MATEMATIKA 10

ALGEBRA VA ANALIZ ASOSLARI
GEOMETRIYA
II QISM

O‘rta ta’lim muassasalarining 10-sinfi va o‘rta maxsus, kasb-hunar ta’limi muassasalari o‘quvchilari uchun darslik

1-nashri.

O‘zbekiston Respublikasi Xalq ta’limi vazirligi tasdiqlagan

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Darslikning “Algebra va analiz asoslari” bo‘limida ishlatilgan belgilar va ularning talqini:

△	– masalani yechish (isbotlash) boshlandi	▲	– masalani yechish (isbotlash) tugadi
!	– nazorat ishlari va test (sinov) mashqlari	?	– savol va topshiriqlar
	– asosiy ma’lumot		– murakkabroq mashqlar

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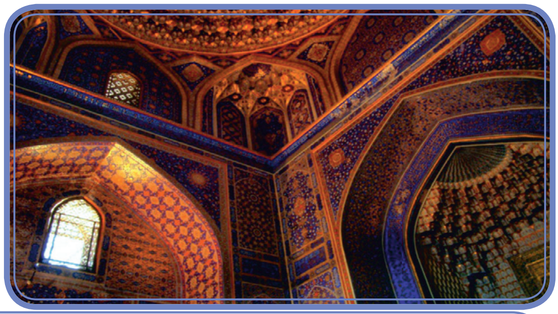


III BOB

ELEMENTAR FUNKSIYALAR VA TENGLAMALAR

47-49 MUNOSABATLAR VA AKSLANTIRISHLAR. FUNKSIYA

Quyidagi jadvalda Nyu York shahrining aeroportida avtomashinalar turargohida vaqtga qarab to‘lanishi lozim bo‘lgan mablag‘ miqdorlari keltirilgan:

Ravshanki, to‘lanadigan mablag‘ qiymati vaqt davomiyligiga bevosita bog‘liq.

Vaqt (t)	Qiymati
0 – 1 soat	$5,00
1 – 2 soat	$9,00
2 – 3 soat	$11,00
3 – 6 soat	$13,00
6 – 9 soat	$18,00
9 – 12 soat	$22,00
12 – 24 soat	$28,00

Bu jadvalga qarab quyidagi savolga javob beraylik:

Avtomashinaning aynan bir soat turishi uchun qancha pul sarflanadi?

5 AQSh dollarimi, 9 AQSh dollarimi yoki 11 AQSh dollarimi?

Noqulay vaziyatga tushmaslik va muammoni aniqlashtirish uchun biz jadvaldagi ma’lumotlarni grafik ko‘rinishiga keltiramiz. Jadvaldagi “2 – 3 soat” yozuv “2 soatdan ortiq ammo 3 soatdan ortiqmas vaqt”, ya’ni $2 < t \leqslant 3$ oraliq deb tushuniladi. U holda quyidagi jadvalni hosil qilamiz:

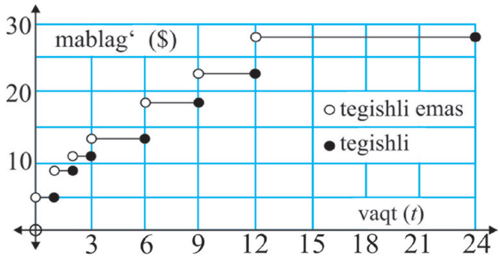
Vaqt (t)	**Qiymati**
$0 < t \leqslant 1$ soat	$5,00
$1 < t \leqslant 2$ soat	$9,00
$2 < t \leqslant 3$ soat	$11,00
$3 < t \leqslant 6$ soat	$13,00
$6 < t \leqslant 9$ soat	$18,00
$9 < t \leqslant 12$ soat	$22,00
$12 < t \leqslant 24$ soat	$28,00

Matematika tilida mazkur jadval ikkita o‘zgaruvchi (*vaqt* va to‘lanadigan *mablag‘ miqdori*) orasidagi **munosabatga** misol bo‘la oladi.

Munosabat tartiblangan juftliklar to‘plami sifatida talqin qilinishi mumkin, masalan

$\{(1, 5), (-2, 3), (4, 3), (1, 6)\}$.

Avtomashinalar turargohida $0 < t \leqslant 24$ oraliqdagi t vaqtga qarab to‘lanishi lozim bo‘lgan mablag‘ o‘zgarishi quyidagicha tasvirlanadi:



Gorizontal o‘qdagi o‘zgaruvchining qabul qiladigan qiymatlar to‘plami munosabatning *aniqlanish sohasi* deyiladi.

Masalan, $\{t|0<t\leqslant 24\}$ to‘plam yuqoridagi *vaqt* va to‘lanadigan *mablag‘ miqdori* orasidagi munosabatning, $\{-2, 1, 4\}$ to‘plam esa {(1, 5), (− 2, 3), (4, 3), (1, 6)} munosabatning aniqlanish sohalari bo‘ladi.

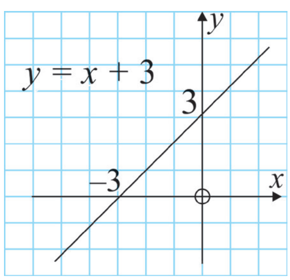
Vertikal o‘qdagi o‘zgaruvchining qabul qiladigan qiymatlar to‘plami munosabatning *qiymatlar to‘plami* deyiladi.

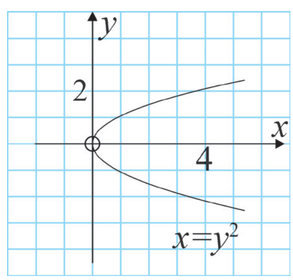
Masalan, {5, 9, 11, 13, 18, 22, 28} to‘plam yuqoridagi *vaqt* va to‘lanadigan mablag‘ *miqdori* orasidagi munosabatning, {3, 5, 6} to‘plam esa {(1, 5), (−2, 3), (4, 3), (1, 6)} munosabatning qiymatlar to‘plamlari bo‘ladi.

Endi munosabatga aniqroq ta’rif beraylik. Dekart koordinatalar tekisligida berilgan nuqtalar to‘plami **munosabat** deyiladi. Ko‘pincha munosabat x, y **o‘zgaruvchilar** qatnashgan tenglama ko‘rinishida beriladi.

Masalan, $y=x+3$, $x=y^2$ tenglamalarning har biri munosabatni aniqlaydi.

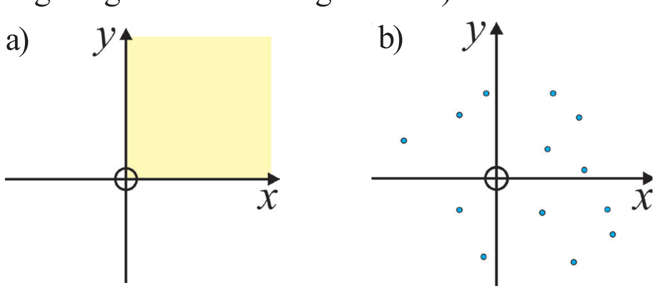
Bu tenglamalarning har biri Dekart koordinatalar tekisligida nuqtalar to‘plamini hosil qiladi.





Ayrim munosabatlarni tenglamalar yordamida yozib bo‘lmaydi.

Masalan, $x>0$, $y>0$ shartni qanoatlantiradigan $(x,\ y)$ nuqtalar to‘plami (koordinatalar tekisligining birinchi choragi *a*-rasm)



yoki ushbu nuqtalar to‘plamini (*b*- rasm) tenglamalar yordamida yozib bo‘lmaydi.

Agar munosabatda birinchi koordinatasi teng bo‘lgan ikkita turli nuqta mavjud bo‘lmasa, bu munosabat **akslantirish** yoki **funksiya** deyiladi.

Demak, funksiya – munosabatning maxsus turi ekan.

Berilgan munosabat funksiya ekanligini tekshirishning ikki usulini keltiramiz.

Algebraik usul

Bu usul munosabat tenglama yordamida berilgan hollarda qo‘llaniladi. Bunda berilgan tenglamaga x va y ning ixtiyoriy qiymatini qo‘yganda x ning har bir qiymati uchun y ning yagona qiymati hosil bo‘lsa, bunday munosabat funksiya bo‘ladi.

Masalan, $y=3x-2$ tenglamaga x ning ixtiyoriy qiymatini qo‘ysak, y ning yagona qiymati hosil bo‘ladi. Demak, bu tenglama yordamida aniqlangan munosabat funksiya bo‘ladi.

Shu bilan birga $x=y^2$ tenglama bilan aniqlangan munosabat funksiya bo‘lmaydi, chunki, masalan, $x=4$ qiymatini qo‘ysak, ikkita $y=\pm2$ qiymat hosil bo‘ladi.

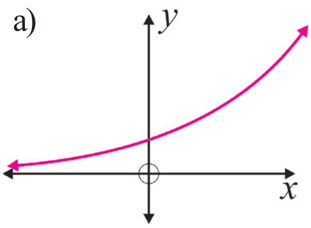
Grafik usul

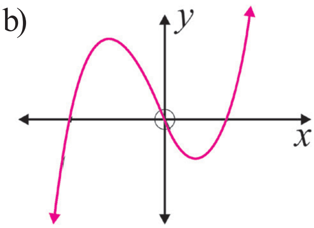
Munosabat Dekart koordinatalar sistemasida to‘plam ko‘rinishida berilgan bo‘lsin. Agar biz barcha mumkin bo‘lgan vertikal to‘g‘ri chiziqlarni chizsak, bu to‘g‘ri chiziqlardan ixtiyoriysining berilgan munosabat bilan kesishish nuqtalari soni bittadan oshmasa, u holda bu munosabat funksiya bo‘ladi. Aksincha, agar qandaydir vertikal to‘g‘ri chiziqning berilgan munosabat bilan kesishish nuqtalari soni bittadan ko‘p bo‘lsa, u holda munosabat funksiya bo‘lmaydi.

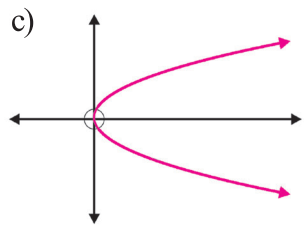
Bunda biz quyidagilarga shartli ravshida kelishamiz:

- Agar chiziqda kichik oq rangdagi doiracha belgilangan bo‘lsa, (—o—), bunday nuqta chiziqqa tegishli emas.
- Agar chiziqda kichik qora rangdagi doiracha belgilangan bo‘lsa, (——●), bu nuqta chiziqqa tegishli.
- ⟶ ko‘rinishdagi (strelka) o‘q chiziq shu yo‘nalishda cheksiz davom ettirilishi mumkinligini bildiradi.

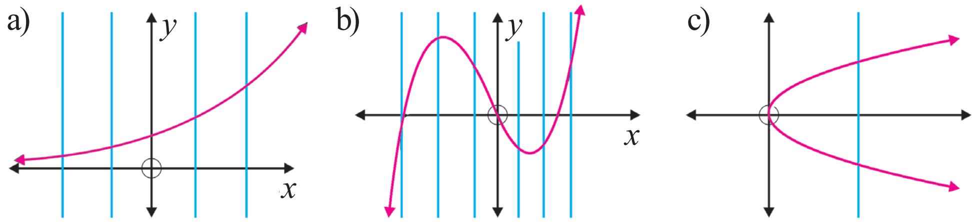
1- misol. Quyidagi munosabatlardan qaysi biri funksiya bo‘lishini tekshiraylik:

a)


b)


c)


△ Vertikal to‘g‘ri chiziqlarni chizib, quyidagi xulosaga kelamiz:



a) va b) munosabatlardan har biri funksiya bo‘ladi (chunki ixtiyoriy vertikal to‘g‘ri chiziq u bilan eng ko‘pi bitta nuqtada kesishadi), c) munosabat esa funksiya emas, chunki uni ikkita nuqtada kesuvchi vertikal to‘g‘ri chiziq mavjud. ▲

Hisoblash uskunasi (moslamasi) quyidagi algoritm bo‘yicha ishlasin:

1- qadam. Biror son kiritilmoqda.

2- qadam. Kiritilgan son 2 ga ko‘patirilmoqda.

3- qadam. Natijaga 3 qo‘shilmoqda.

Masalan, uskunaga 4 soni kiritilsa, natijada $4\cdot2+3=11$ soni hosil bo‘ladi.

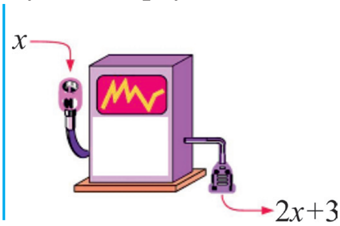
Xuddi shunday uskunaga (-4) soni kiritilsa, natijada $2\cdot(-4)+3=-5$ soni hosil bo‘ladi.

Umumiy holda, uskunaga x soni kiritilsa, natijada yagona $2x+3$ soni hosil bo‘ladi.

Ko‘rinib turibdiki, uskunaga qandaydir x son kiritilsa, natijada yagona $2x+3$ qiymat, hosil bo‘ladi.

Demak, mazkur uskuna ishlaydigan algoritm funksiyani aniqlaydi.

Bu holat $f{:}x\mapsto2x+3$, $f(x)=2x+3$ yoki $y=2x+3$ kabi yoziladi.



Agar $f(x)=2x+3$ bo‘lsa, uning -4 soniga mos qiymati $f(-4)=2(-4)+3=-5$ kabi topiladi.

Umumiy holda, $f(x)$ − funksiyaning berilgan x dagi *qiymati* deb yuritiladi va mazkur munosabat $y=f(x)$ kabi yoziladi.

2- misol. Agar $f{:}x\mapsto 2x^2-3x$ bo‘lsa: a) $f(5)$; b) $f(-4)$ qiymatlarni toping.

△ $f(x)=2x^2-3x$ munosabatga $x=5$ va $x=-4$ sonlarni qo‘yib, ularga mos qiymatlarni topamiz: a) $f(5)=2\cdot(5)^2-3\cdot(5)=2\cdot25-15=35$;

b) $f(-4)=2\cdot(-4)^2-3\cdot(-4)\ =2\cdot(16)+12=44$. ▲

3- misol. Agar $f(x)=5-x-x^2$ bo‘lsa: a) $f(-x)$; b) $f(x+2)$ qiymatlarni toping va natijalarni soddalashtiring.

△ $f(x)=5-x-x^2$ funksiyaga x o‘rniga $-x$ va $x+2$ qiymatlarni qo‘yib, ularga mos qiymatlarni topamiz:

a) $f(-x)=5-(-x)-(-x)^2=5+x-x^2$;

b) $f(x+2)=5-(x+2)-(x+2)^2=5-x-2-[x^2+4x+4]=3-x-x^2-4x-4=$
$=-x^2-5x-1.$ ▲

Savol va topshiriqlar

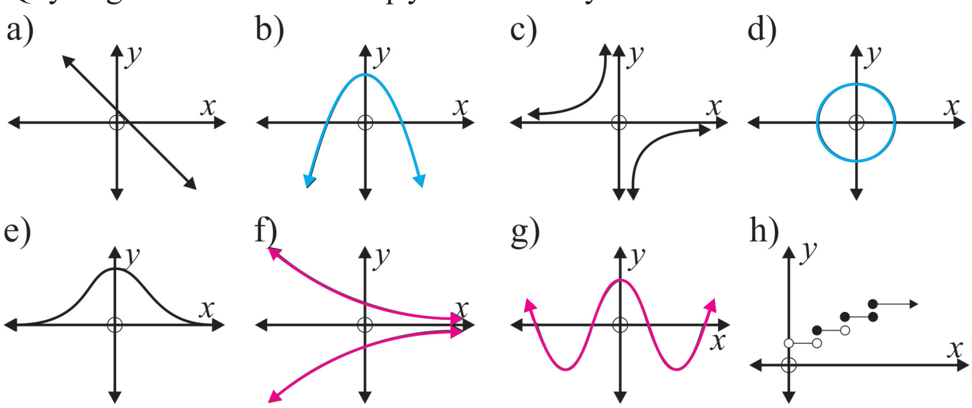
1. Munosabatga misollar keltiring.
2. Akslantirish yoki funksiyaga ta’rif bering.
3. Funksiyaninig aniqlanish sohasini tushuntiring.
4. Funksiyaninig qiymatlar sohasini tushuntiring.

Mashqlar

73. Quyidagi munosabatlardan qaysilari funksiya bo‘ladi:

a) $\{(1, 3), (2, 3), (3, 5), (4, 6)\}$;
b) $\{(1, 3), (3, 2), (1, 7), (-1, 4)\}$;
c) $\{(2, -1), (2, 0), (2, 3), (2, 11)\}$;
d) $\{(7, 6), (5, 6), (3, 6), (-4, 6)\}$;
e) $\{(0, 0), (1, 0), (3, 0), (5, 0)\}$;
f) $\{(0, 0), (0, -2), (0, 2), (0, 4)\}$?

74. Quyidagi munosabatlardan qaysilari funksiya bo‘ladi?



75. Dekart koordinatalar tekisligida berilgan har qanday to‘g‘ri chiziq funksiya bo‘ladimi? Javobingizni asoslang.

76. $x^2+y^2=9$ tenglama yordamida berilgan munosabat funksiya bo‘ladimi?

77. Agar $f{:}x\mapsto 3x+2$ bo‘lsa, quyidagi qiymatlarni toping:

A) $f(0)$; B) $f(2)$; C) $f(-1)$; D) $f(-5)$; E) $f\left(-\frac{1}{3}\right)$.

78. Agar $f{:}x\mapsto 3x-x^2+2$ bo‘lsa, quyidagi qiymatlarni toping:

A) $f(0)$; B) $f(3)$; C) $f(-3)$; D) $f(-7)$; E) $f\left(\frac{2}{3}\right)$.

79. Agar $g: x\mapsto x-\frac{4}{x}$ bo‘lsa, quyidagi qiymatlarni toping:

A) $g(1)$; B) $g(4)$; C) $g(-1)$; D) $g(-4)$; E) $g\left(-\frac{1}{2}\right)$.

80. Agar $f(x)=7-3x$ bo‘lsa, quyidagi qiymatlarni toping va natijani soddalashtiring.

a) $f(a)$; | b) $f(-a)$; | c) $f(a+3)$; | d) $f(b-1)$; | e) $f(x+2)$; | f) $f(x+h)$.

81. Agar $F(x)=2x^2+3x-1$ bo‘lsa, quyidagi qiymatlarni toping va natijani soddalashtiring.

a) $F(x+4)$; | b) $F(2-x)$; | c) $F(-x)$; | d) $F(x^2)$; | e) $F(x^2-1)$; | f) $F(x+h)$.

82. $G(x)=\dfrac{2x+3}{x-4}$ funksiya uchun:

a) **I** $G(2)$ **II** $G(0)$ **III** $G\left(-\dfrac{1}{2}\right)$ larni toping;

b) Qanday x larda $G(x)$ mavjud emas?

c) $G(x+2)$ ni toping va soddalashtiring;

d) x ning $G(x)=-3$ bo‘ladigan x qiymatini toping.

83. Funksiya f harfi bilan belgilangan bo‘lsin. f va $f(x)$ belgilarning ma’nolari orasida qanday farq bor?

84. Eskirish natijasida nusxa ko‘paytirish uskunasining t yildan so‘ng narxi $V(t)=9650-860t$ qonuniyat bo‘yicha o‘zgaradi.

a) $V(4)$ ni toping va uning ma’nosini tushuntiring;

b) $V(t)=5780$ bo‘lganda t ni toping. Vaziyatni tushuntiring;

c) Uskuna qaysi narxda sotib olingan?

85. Bitta koordinatalar tekisligida $f(2)=1$, $f(5)=3$ bo‘ladigan uchta turli funksiya grafiklarini chizing.

86. $f(2)=1$ va $f(-3)=11$ bo‘ladigan $f(x)=ax+b$ chiziqli funksiyani toping.

87. $f(x)=ax+\dfrac{b}{x}$, $f(1)=1$, $f(2)=5$ bo‘lsa, a, b larni toping.

88. $T(0)=-4$, $T(1)=-2$, $T(2)=6$ bo‘ladigan $T(x)=ax^2+bx+c$ kvadrat funksiyani toping.

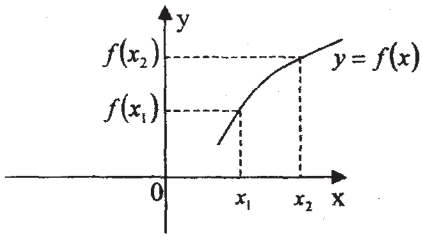
89. $f(x)=2^x$ bo‘lsa, $f(a)f(b)=f(a+b)$ tenglikni isbotlang.

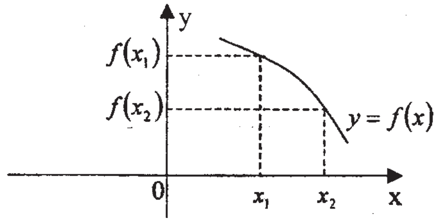
50-51 ELEMENTAR FUNKSIYALARNING MONOTONLIGI, ENG KATTA VA ENG KICHIK QIYMATLARI HAQIDA TUSHUNCHA

Funksiyaning monotonligi

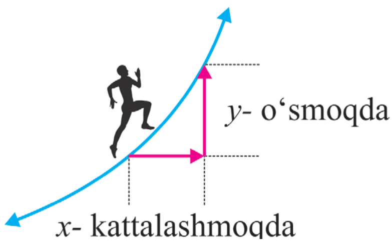
Agar $x_1<x_2$ tengsizlikni qanoatlantiruvchi barcha $x_1, x_2 \in I$ uchun $f(x_1)<f(x_2)$ tengsizlik o‘rinli bo‘lsa, I oraliqda $y=f(x)$ funksiya *o‘suvchi* deyiladi.

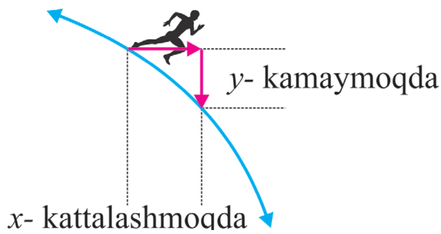
Agar $x_1<x_2$ tengsizlikni qanoatlantiruvchi barcha $x_1, x_2 \in I$ uchun $f(x_2)<f(x_1)$ tengsizlik o‘rinli bo‘lsa, I oraliqda $y=f(x)$ funksiya *kamayuvchi* deyiladi.



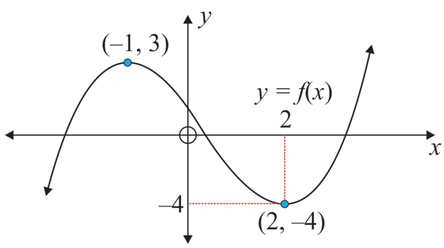


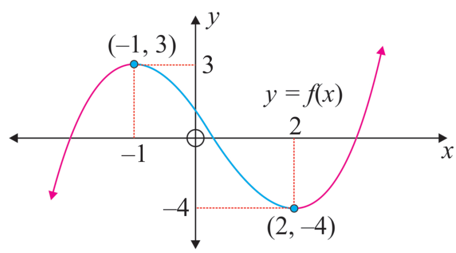
Agar funksiya o‘suvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga “harakat” qilsak, ordinatalar ortadi; funksiya kamayuvchi bo‘lsa, ordinatlar kamayadi.





1- misol. Funksiyaning o‘sish va kamayish oraliqlarini toping:





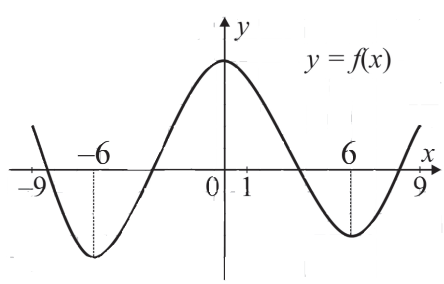
△Agar funksiya o‘suvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga harakat qilsak, ordinatalar o‘sadi (grafikda qizil rangda ajratilgan). Demak funksiya $x \leqslant -1$ va $x \geqslant 2$ oraliqlarda o‘sadi. Javobni $(-\infty, -1] \cup [2, +\infty)$ ko‘rinishda ham yozsa bo‘ladi.

Xuddi shunday, agar funksiya kamayuvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga harakat qilsak, ordinatalar kamayadi (grafikda ko‘k rangda ajratilgan). Demak, funksiya $-1 \leqslant x \leqslant 2$ oraliqlarda kamayadi. ▲

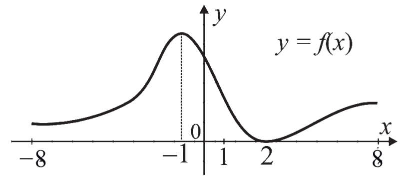
2- misol. Funksiya qaysi oraliqlarda o‘sadi?

△Bu funksiya $[-9; 9]$ oraliqda berilgan.

Agar funksiya o‘suvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga harakat qilsak, ordinatalar kattalashadi. Demak, funksiya $[-6;0]$ va $[6;9]$ oraliqlarda o‘sadi. Javobni $[-6;0] \cup [6;9]$ ko‘rinishda ham yozsa bo‘ladi. ▲

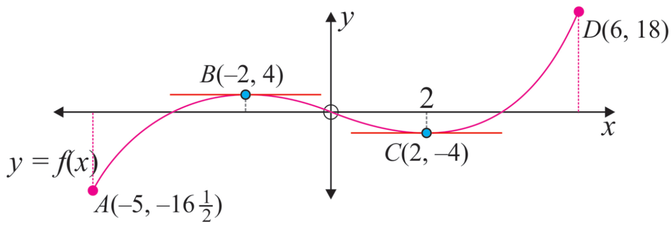


3- misol. Funksiya qaysi oraliqlarda kamayadi?



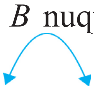
△ Agar funksiya kamayuvchi bo‘lsa, grafik bo‘ylab chapdan o‘ngga harakat qilsak, ordinatalar kichiklashadi. Demak funksiya $[-1;\ 2]$ oraliqda kamayadi. ▲

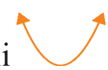
Funksiyaning eng katta va eng kichik qiymatlari haqida tushuncha beramiz. $-5 \leqslant x \leqslant 6$ oraliqda aniqlangan funksiya grafigini qaraylik.



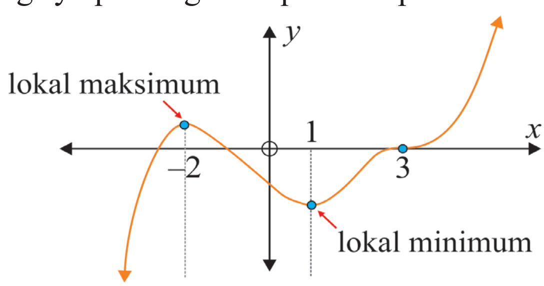
A nuqtaning ordinatasi boshqa nuqtalar ordinatalaridan kichik bo‘lgani sababli shu nuqta **global minimum** nuqtasi deyiladi. Funksiyaning unga mos bo‘lgan qiymati **funksiyaning eng kichik qiymati** deyiladi. Bizning misolimizda funksiyaning eng kichik qiymati $-16{,}5$ ga teng.

Xuddi shunday, *D* nuqtaning ordinatasi boshqa nuqtalar ordinatalaridan katta bo‘lgani sababli shu nuqta **global maksimum** nuqtasi deyiladi. Funksiyaning unga mos bo‘lgan qiymati **funksiyaning eng katta qiymati** deyiladi. Bizning misolimizda funksiyaning eng katta qiymati 18 ga teng.

Endi *B* nuqtaga e’tibor beraylik. Grafikning unga yaqin bo‘lgan nuqtalari to‘plami  shaklga ega. Bunday hossaga ega bo‘lgan nuqta **lokal maksimum** nuqtasi deyiladi.

Huddi shunday, grafikning *C* nuqtaga yaqin bo‘lgan nuqtalari to‘plami  shaklga ega. Bunday hossaga ega bo‘lgan nuqta **lokal minimum** nuqtasi deyiladi.

Faqat lokal minimum va lokal maksimumga ega bo‘lgan funksiyaga misol keltiraylik:



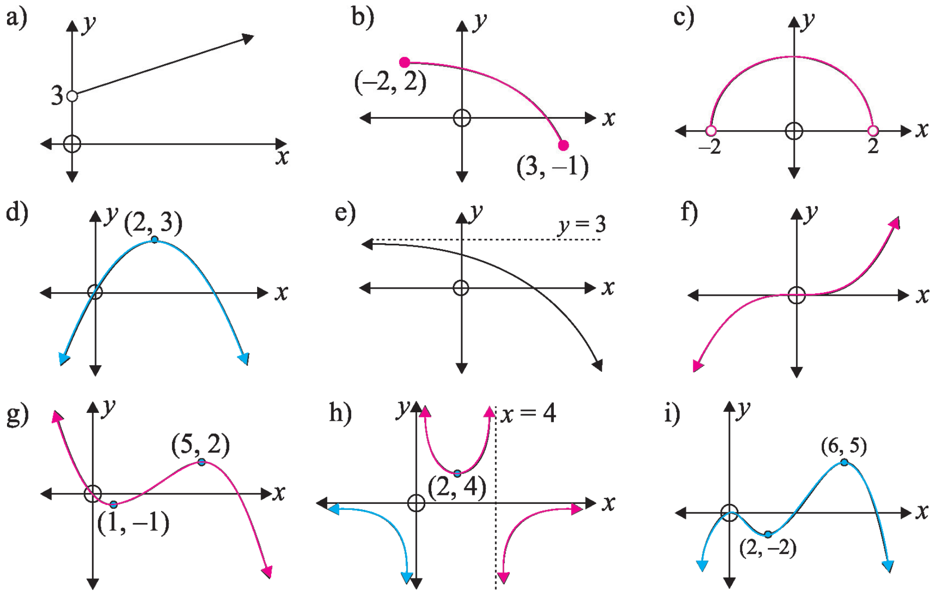
Savol va topshiriqlar

1. Oraliqda o‘suvchi funksiyaga ta'rif bering;
2. Oraliqda kamayuvchi funksiyaga ta'rif bering;

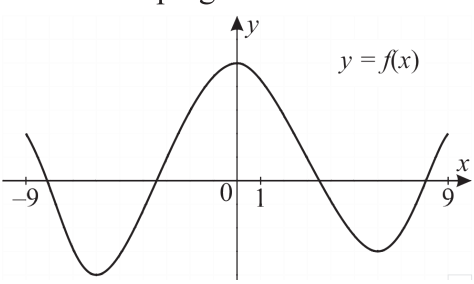
3. Chizmaga qarab funksiyaning o‘sishi qanday aniqlanadi?
4. Chizmaga qarab funksiyaning kamayishi qanday aniqlanadi?

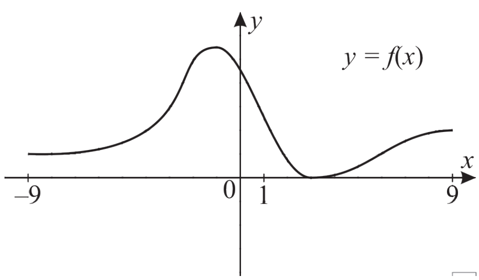
Mashqlar

90. Funksiya uchun quyidagi oraliqlarni toping: 1) o‘sish; 2) kamayish. Agar mumkin bo‘lsa, ularning lokal maksimumini va minimumini, eng katta va eng kichik qiymatlarini toping.



91. [–9; 9] oraliqda berilgan funksiya qaysi oraliqlarda o‘sadi? Qaysi oraliqlarda kamayadi? Uning lokal maksimumini va minimumini, eng katta va eng kichik qiymatlarini toping:



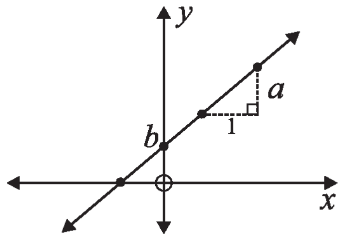


52-54 CHIZIQLI VA KVADRATIK MODELLAR

Chiziqli funksiya

$f(x)=ax+b$ koʻrinishdagi funksiya chiziqli deyiladi, bu yerda x, y – oʻzgaruvchilar, a, b – berilgan sonlar, $a\neq 0$.

Chiziqli funksiya grafigi koordinata tekisligida toʻgʻri chiziq boʻlib, bunda a son burchak koeffitsienti deyiladi.



Quyida biz chiziqli funksiya tadbiqlarini keltiramiz.

1- misol. Tennis kortini ijaraga olish narxi $C(h)=5h+8$ (AQSh dollari) formula bilan aniqlangan, bu yerda h – ijara vaqti (soatda). 4 soat va 10 soat uchun ijaraga qancha mablagʻ sarflanadi?

△ $C(h)=5h+8$ formuladan foydalanib, $C(4)=5\cdot 4+8=20+8=28$ va $C(10)=5\cdot 10+8=50+8=58$ ekanligini topamiz. Demak, 4 soatga 28 AQSh dollari, 10 soatga esa 58 AQSh dollari mablagʻ sarflanadi. ▲

2- misol. Nyu Yorkda taksi passajir olish uchun toʻxtashga 3 AQSh dollari 30 sent, har kilometrga esa 1 AQSh dollari 75 sent oladi.

a) Jadvalni daftaringizga koʻchirib oling va uni toʻldiring:

d – masofa (km)	0	2	4	6	8	10
C – mablagʻ ($)						

b) C va d orasidagi bogʻlanishni grafik koʻrinishda ifodalang;

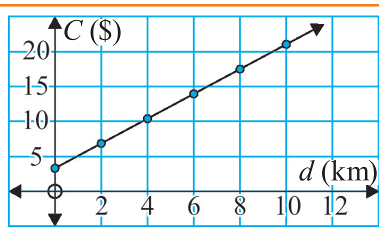
c) C (d) funksiyaning algebraik koʻrinishini–formulasini yozing;

d) 9,4 km yurish uchun qancha mablagʻ sarflanadi?

△ a) 3,3 AQSh dollariga ketma-ket $2\cdot 1,75=3,5$ AQSh dollarini qoʻshib, kataklarni toʻldiramiz:

d – masofa (km)	0	2	4	6	8	10
C – mablagʻ ($)	3,30	6,80	10,30	13,80	17,30	20,80

b)



Bu – chiziqli funksiya.

c) Burchak koeffitsiyentini topamiz:

$$a=\frac{20,80-17.30}{10-8}=1,75.$$

Demak, $C(d)=1,75d+3,3$.

d) $C(9,4)=1,75\cdot 9,4+3,3=19,75$.

Demak, 19,75 AQSh dollari sarflanadi. ▲

Kvadrat funksiya

$y=ax^2+bx+c$ ko‘rinishdagi funksiya kvadrat fuksiya deyiladi, bu yerda x, y – o‘zgaruvchilar, a, b, c – berilgan sonlar, $a \neq 0$.

$y=2x^2+4x-5$ funksiyaning a) $x=0$; b) $x=3$ nuqtalardagi qiymatini topaylik.

a) $x=0$ bo‘lsin. U holda $y=2\cdot0^2+4\cdot0-5=0+0-5=-5$.

b) $x=3$ bo‘lsin. U holda $y=2\cdot3^2+4\cdot3-5=18+12-5=25$.

3- misol. Tosh otilganda t sekundda uning yerga nisbatan balandligi $h(t)=-5t^2+30t+2$ funksiya yordamida aniqlanadi.

a) $t=3$ bo‘lganda tosh yerdan qancha balandda bo‘ladi?

b) Tosh qanday balandlikdan turib otildi?

c) Qaysi vaqtda toshning balandligi 27 metr bo‘ladi?

△ a) $h(3)=-5\cdot3^2+30\cdot3+2=-45+90+2=47$. Demak, otilgan tosh t=3 sekunddan so‘ng 47 metr balandlikda bo‘ladi.

b) tosh $t=0$ bo‘lganda otilgani bois, $h(0)=-5\cdot0^2+30\cdot0+2=2$. Demak, tosh 2 metr balandlikdan otilgan.

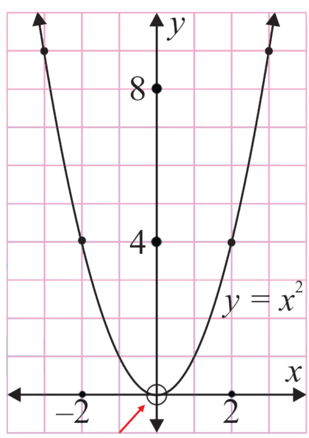
c) Tosh yerdan 27 metr balandlikda bo‘lsa, $h(t)=27$ bo‘ladi, ya’ni $-5t^2+30t+2=27$. Bu tenglamani yechamiz: $-5t^2+30t-25=0$, $t^2-6t+5=0$, $t_1=1$, $t_2=5$. Demak, tosh 27 metr balanlikda 1 sekunddan so‘ng (tepaga ko‘tarilayotganda) va 5 sekunddan so‘ng (pastga tushayotganda) bo‘ladi. ▲

Kvadrat funksiya grafigi

$f(x)=x^2$ funksiyani qaraylik. Uning ba’zi nuqtalardagi qiymatlari jadvalini tuzamiz:

x	-3	-2	-1	0	1	2	3
$f(x)$	9	4	1	0	1	4	9

Shu jadvaldagi (x, y) nuqtalarni koordinata tekisligida yasab, ularni silliq chiziq bilan tutashtirib, ushbu grafikni hosil qilamiz:



Hosil bo‘lgan shakl **parabola** deb ataladi. Ko‘rinib turibdiki, parabola tarmoqlari yuqoriga yo‘nalgan bo‘lib, u ordinata o‘qiga nisbatan simmetrik bo‘lgan egri chiziqdir.

(0, 0) nuqta $y=x^2$ **parabolaning uchi** deyiladi.

4-misol. $y=x^2-2x-5$ kvadrat funksiya grafigini yasang.

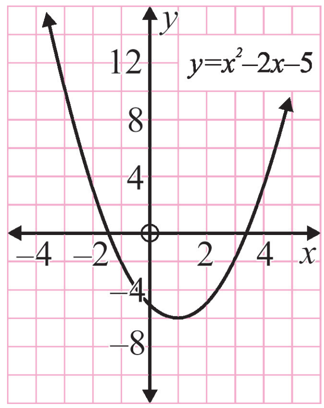
△ Funksiyaning bitta nuqtadagi, masalan $x=-3$ nuqtasidagi qiymatini topaylik:

$f(-3)=(-3)^2-2(-3)-5=9+6-5=10$.

Funksiyaning bir nechta nuqtadagi qiymatini topib, jadvalni tuzamiz:

x	-3	-2	-1	0	1	2	3
y	10	3	-2	-5	-6	-5	-2

(x, y) nuqtalarni koordinata tekisligida yasab, ularni silliq chiziq bilan tutashtirib, berilgan kvadrat funksiya grafigini hosil qilamiz:



Hosil boʻlgan grafik ham parabola shaklida. Uning tarmoqlari esa yuqoriga yoʻnalgan. ▲

Ihtiyoriy $y=ax^2+bx+c$ parabolaning ordinatalar oʻqi – Oy oʻqi bilan kesishish nuqtasini topamiz:

$x=0$, $y=a\cdot 0^2+b\cdot 0+c=0+0+c=c$.

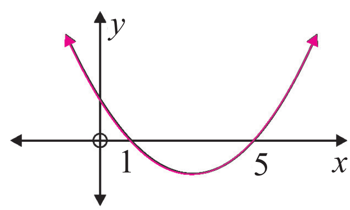
Demak, parabola $(0, c)$ nuqtada ordinatalar oʻqi bilan kesishadi.

$y=ax^2+bx+c$ parabolaning abssissalar oʻqi bilan kesishish nuqtalarini topish uchun $ax^2+bx+c=0$ kvadrat tenglamaning yechimlarini topish kifoya.

Masalan, $y=x^2-2x-15$ parabolaning abssissalar oʻqi bilan kesishish nuqtalarini topamiz. $x^2-2x-15=0$ deb, bu kvadrat tenglamani yechamiz. Uning yechimlari $x=-3$ va $x=5$ boʻladi. Demak, $y=x^2-2x-15$ parabola abssissalar oʻqi bilan $(-3, 0)$, $(5, 0)$ nuqtalarda kesishishadi. $y=ax^2+bx+c$ parabola uchun $x=h$ koʻrinishdagi vertikal toʻgʻri chiziq uning simmetriya oʻqi boʻladi.

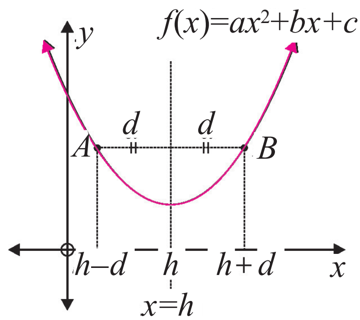
Agar $y=ax^2+bx+c$ parabola abssissalar oʻqi bilan kesishsa, h son parabolaning Ox oʻqi bilan kesishish nuqtalari abssissalarining oʻrta arifmetigiga teng boʻladi.

5- misol. Rasmdagi parabolaning simmetriya oʻqini toping.



△ Agar parabola abssissalar oʻqi bilan $(1, 0)$ va $(5, 0)$ nuqtalarda kesishsa, $x=\frac{5+1}{2}=3$ - simmetriya oʻqi boʻladi. ▲

Agar $y=ax^2+bx+c$ parabola abssissalar oʻqi bilan kesishmasa, h sonni boshqa usulda ham topsa boʻladi.



Koʻrinib turibdiki, abssissalari $h-d$ va $h+d$ boʻlgan A, B nuqtalar bir hil ordinatalarga ega, yaʼni $f(h-d)=f(h+d)$, demak, A va B $x=h$ oʻqqa nisbatan simmetrik nuqtalardir.

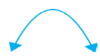
Bu shartdan foydalanib quyidagi tenglikdan h ni topamiz:

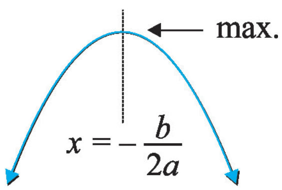
$a(h-d)^2+b(h-d)+c=a(h+d)^2+b(h+d)+c$ yoki

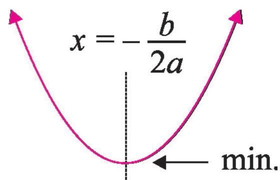
$a(h^2-2hd+d^2)+bh-bd=a(h^2-2hd+d^2)+bh+bd$ yoki $-\ 4ahd=2bd.$

Demak, simmetriya o‘qi $h=\dfrac{-b}{2a}$ ekan.

Xulosa. $y=ax^2+bx+c$ parabolaning simmetriya o‘qi $x=\dfrac{-b}{2a}$ ko‘rinishda bo‘ladi. Parabolaning o‘z-o‘ziga simmetrik bo‘lgan nuqtasi parabolaning uchi deyiladi. Parabola uchining koordinatalari $x=\dfrac{-b}{2a}$, $y=0$. Parabola o‘qi $(-\dfrac{b}{2a},0)$ nuqtadan Oy o‘qiga paralel bo‘lib o‘tadi. Parabola uchi simmetriya o‘qiga tegishli bo‘lgani bois, uning abssissasi $\dfrac{-b}{2a}$ ga teng.

Ravshanki, $a<0$ bo‘lganda parabola shakli  kabi bo‘lib, uning uchi $y=ax^2+bx+c$ kvadrat funksiyaning maksimum nuqtasi, $a>0$ bo‘lganda parabola shakli  kabi bo‘lib, uning uchi kvadrat funksiyaning minimum nuqtasi bo‘ladi.





6- misol. $y=3x^2+4x-5$ parabolaning simmetriya o‘qini toping.

△ $y=3x^2+4x-5$ uchun $a=3,\ b=4.$

Demak, $x=\dfrac{-b}{2a}=\dfrac{-4}{2\cdot3}=-\dfrac{2}{3}$, ya’ni $x=-\dfrac{2}{3}$ – simmetriya o‘qi. ▲

7- misol. $f(x)=x^2+6x+4$ parabolaning uchini toping.

△ $a=1,\ b=6.\quad x=\dfrac{-b}{2a}=\dfrac{-6}{2\cdot1}=-3.$

Demak, parabola uchining abssissasi $x=-3$,

ordinatasi esa: $y=f(-3)=(-3)^2+6(-3)+4=9-18+4=-5.$

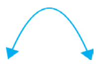
Shuning uchun, parabola uchi $(-3,-5)$ koordinatalarga ega. ▲

8-misol. Sportchi to‘pni yuqoriga otdi, bunda to‘pning t sekunddan keyingi balandligi $H(t)=30t-5t^2$ metr bo‘ldi, $t\geqslant0$.

a) Eng yuqori nuqtaga to‘p necha sekundda yetadi?

b) Eng yuqori nuqta yerdan qancha balandlikda bo‘ladi?

c) To‘p necha sekunddan keyin yerga tushadi?

△ a) $H(t)=30t-5t^2$ uchun $a<0$, $a=-5$. Shuning uchun bu parabola quyidagi shaklda bo‘ladi: . $t=\frac{-b}{2a}=\frac{-30}{2\cdot(-5)}=3$ sekundda maksimumga erishiladi.

Ya’ni eng yuqori nuqtaga to‘p 3 sekundda ko‘tariladi.

b) Maksimal balandlikni topamiz:

$H(3)=30\cdot3-5\cdot3^2=90-45=45$, ya’ni eng yuqori nuqta yerdan 45 metr balandlikda bo‘ladi.

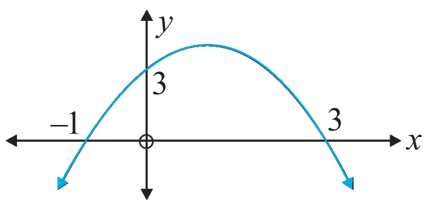
c) $H(t)=0$ bo‘lsa, to‘p yerga tushadi. Shu tenglamani yechamiz:

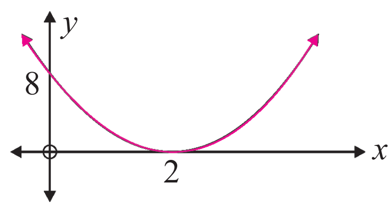
$30t-5t^2=0$, $5t^2-30t=0$, $5t(t-6)=0$. Bundan $t_1=0$ yoki $t_2=6$.

Demak, 6 sekunddan keyin to‘p yerga tushadi. ▲

Quyida biz parabola shakliga qarab kvadrat funksiya formulasini topishga doir misollar keltiramiz.

9- misol. Berilgan parabolalarga qarab kvadrat funksiya formulasini yozing:

a) 

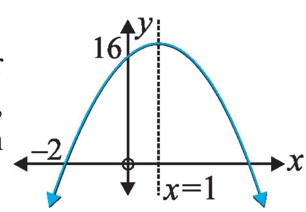
b) 

△ a) Parabola tarmoqlari pastga qaragan, u abssissalar o‘qi bilan -1 va 3 nuqtalarda kesishadi. Shuning uchun $y=a(x+1)(x-3)$, $a<0$. $x=0$ da $y=3$ shartdan $a=-1$ ni topamiz.

Demak, kvadrat funksiya $y=-(x+1)(x-3)=-x^2+2x+3$ formula bilan ifodalanadi.

b) Parabola tarmoqlari yuqoriga qaragan, u abssissalar o‘qiga $x=2$ nuqtada urinadi. Shuning uchun $y=a(x-2)^2$, $a>0$. $x=0$ da $y=8$ shartdan $a=2$ ni topamiz. Demak, kvadrat funksiya $y=2(x-2)^2$ formula bilan beriladi. ▲

10- misol. Berilgan parabolaga qarab kvadrat funksiya formulasini yozing.



△ $x=1$ – simmetriya o‘qi bo‘lgani sababli, abssissalar o‘qi bilan ikkinchi kesishish nuqtasi $x=4$ bo‘ladi. Demak, $y=a(x+2)(x-4)$. Bundan $x=0$, $y=16$. Shuning uchun $16=a(0+2)(0-4)$. Bu yerdan $a=-2$ yoki $y=-2(x+2)\cdot(x-4)=-2x^2+4x+16$. ▲

Savol va topshiriqlar.

1. Chiziqli funksiya nima?
2. Chiziqli funksiyaning burchak koeffitsiyenti nima?
3. Kvadrat funksiya nima?

4. Kvadrat funksiyaning uchi qanday topiladi?
5. Qachon kvadrat funksiya maksimumga ega bo‘ladi?
6. Qachon kvadrat funksiya minimumga ega bo‘ladi?

Mashqlar

92. Eskirishi natijasida avtomashina narxi t yildan so‘ng $V(t)=25000-3000t$ yevro qonuniyat bilan o‘zgaradi.

a) $V(0)$ qiymatni toping. Bu qiymat ma’nosini tushuntiring;

b) $V(3)$ qiymatni toping. Bu qiymat ma’nosini tushuntiring;

c) $V(t)=10000$ qiymatga necha yildan so‘ng erishiladi?

93. AQShda elektr montajchi chaqirilgani uchun \$60 va har bir soat uchun \$45 hizmat haqqini oladi.

a) $t=0, 1, 2, 3, 4, 5$ bo‘lganda mos jadvalni tuzing. C hizmat haqqining t vaqtga qanday bog‘liqligini grafik ko‘rinishda ifodalang.

b) $C(t)$ funksiyaning formulasini (algebraik ko‘rinishini) yozing.

c) $6\frac{1}{2}$ soat vaqt uchun qancha mablag‘ to‘lanadi?

94. Sisterna 265 litr suv bilan to‘ldirilgan. Undan har bir minutda 11 litr suv olinmoqda.

a) $t=0, 1, 2, 3, 4, 5$ bo‘lganda oqib chiqayotgan suvning V litr hajmi t (minut) vaqtga qanday bog‘liqligini ifodalovchi jadval tuzing.

b) $V(t)$ bog‘lanishni grafik ko‘rinishda ifodalang;

c) $V(t)$ funksiyaning formulasini (algebraik ko‘rinishini) yozing.

d) 15 minutdan keyin sisternada qancha suv qoladi?

e) Sisterna qancha vaqtdan keyin bo‘shaydi?

95. Quyidagilardan qaysi biri kvadrat funksiya bo‘ladi:

a) $y=2x^2-4x+10$; c) $y=-2x^2$; e) $3y+2x^2-7=0$;

b) $y=15x-8$; d) $y=\frac{1}{3}x^2+6$; f) $y=15x^3+2x-16$?

96. (x, y) juftlik ko‘rsatilgan $y=ax^2+bx+c$ kvadrat funksiya bilan ifodalangan munosabatda bo‘ladimi:

a) $f(x)=6x^2-10$, $(0, 4)$;
b) $y=2x^2-5x-3$, $(4, 9)$;
c) $y=-4x^2+6x$, $(-\frac{1}{2}, -4)$;
d) $y=-7x^2+9x+11$, $(-1, -6)$;
e) $f(x)=3x^2-11x+20$, $(2, -10)$;
f) $f(x)=-3x^2+x+6$, $(\frac{1}{3}, 4)$?

97. $y=ax^2+bx+c$ kvadrat funksiya uchun y ning berilgan qiymatiga mos bo‘lgan x ning qiymatini toping:

a) $y=x^2+6x+10$, $y=1$; c) $y=x^2-5x+1$, $y=-3$;
b) $y=x^2+5x+8$, $y=2$; d) $y=3x^2$, $y=-3$.

98. Moddiy jism 80 m/s tezlikda balandga otilgan. Uning t sekundda yerga nisbatan balandligi $h(t)=80t-5t^2$ funksiya yordamida aniqlanadi.
a) 1 sekund, 3 sekund, 4 sekunddan keyin jismning balandligini toping;
b) qaysi vaqtda jismning balandligi 140 metr bo'ladi? 0 metr-chi? Javoblarga mos holatlarni tushuntiring;

99. Mahsulot ishlab chiqaruvchi tadbirkorning daromadi quyidagi formula bilan hisoblanadi:

$P(x)=-\frac{1}{2}x^2+36x-40$ (ming so'm), bu yerda x – mahsulotlarning soni.

a) 0 ta mahsulot, 20 ta mahsulot ishlab chiqarilganda tadbirkor qanday daromadga ega bo'ladi? b) 270 ming so'm daromad olish uchun tadbirkor nechta mahsulot ishlab chiqishi kerak?

100. Funksiyalarning $x=-3, -2, -1, 0, 1, 2, 3$ qiymatlarga mos qiymatlarini toping. Natijalarni jadval ko'rinishida bering va grafiklarni yasang:
a) $y=x^2+2x-2$; d) $f(x)=-x^2+x+2$; g) $y=x^2-5x+6$;
b) $y=x^2-3$; e) $y=x^2-4x+4$; h) $y=x^2+x+1$;
c) $y=x^2-2x$; f) $f(x)=-2x^2+3x+10$; i) $y=-x^2+x-1$?

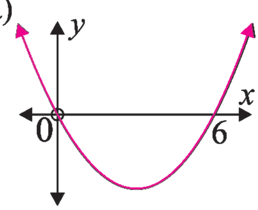
101. Bu grafiklar qanday shaklda bo'ladi?
a) $y=x^2+2x+3$; d) $f(x)=3x^2-10x+1$; g) $y=8-x-2x^2$;
b) $y=2x^2+5x-1$; e) $y=3x^2+5$; h) $f(x)=2x^2-x^2-5$;
c) $y=-x^2-3x-4$; f) $y=4x^2-x$; i) $y=6x^2+2-5x$
parabolalarning ordinatalar o'qi bilan kesishish nuqtasini toping.

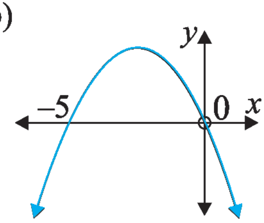
102. Funksiyalar grafiklari ordinatalar o'qi bilan qanday nuqtalarda kesishadi:
a) $y=(x+1)(x+3)$; d) $y=(2x+5)(3-x)$; g) $y=(x-1)(x-6)$;
b) $y=(x-2)(x+3)$; e) $y=x(x-4)$; h) $y=-(x+2)(x+4)$;
c) $y=(x-7)^2$; f) $y=-(x+4)(x-5)$; i) $y=-(x-3)(x-4)$?

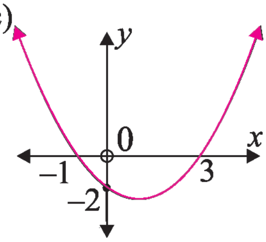
103. a) $y=x^2-x-6$; d) $y=3x-x^2$; g) $y=-x^2-4x+21$; j) $y=-2x^2+x-5$;
b) $y=x^2-16$; e) $y=x^2-12x+36$; h) $y=2x^2-20x+50$; k) $y=-6x^2+x+5$;
c) $y=x^2+5$; f) $y=x^2+x-7$; i) $y=2x^2-7x-15$; l) $y=3x^2+x-1$
parabolalarning abssissalar o'qi bilan kesishish nuqtalarini toping.

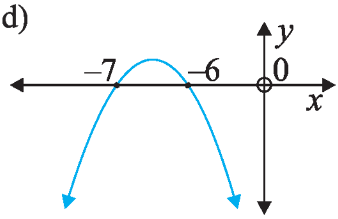
104. a) $y=x^2+x-2$; d) $y=x^2+x+4$; g) $y=-x^2-7x$; j) $y=-x^2+2x-9$;
b) $y=(x+3)^2$; e) $y=3x^2-3x-36$; h) $y=-2x^2+3x+7$; k) $y=4x^2-4x-3$;
c) $y=(x+5)(x-2)$; f) $y=-x^2-8x-16$; i) $y=2x^2-18$; l) $y=6x^2-11x-10$
parabolalarning koordinatalar o'qlari bilan kesishish nuqtalarini toping.

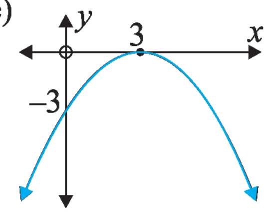
105. Parabolaning simmetriya o‘qini toping:

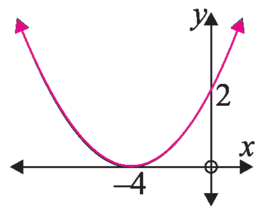
a)


b)


c)


d)


e)


f)


106. Parabolaning simmetriya o‘qini toping:

a) $y=(x-2)(x-6)$;
b) $y=x(x+4)$;
c) $y=-(x+3)(x-5)$;
d) $y=(x-3)(x-8)$;
e) $y=2(x-5)^2$;
f) $y=3(x+2)^2$.

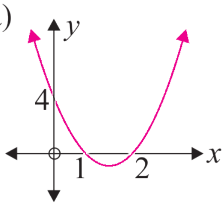
107. Parabolaning simmetriya o‘qini toping:

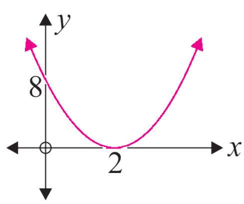
a) $y=x^2+6x+2$;
b) $y=x^2-8x-1$;
c) $f(x)=2x^2+5x-3$;
d) $y=-x^2+3x-7$;
e) $y=2x^2-5$;
f) $y=-5x^2+7x$;
g) $f(x)=x^2-6x+9$;
h) $y=10x-3x^2$;
i) $y=\frac{1}{8}x^2+x-1$.

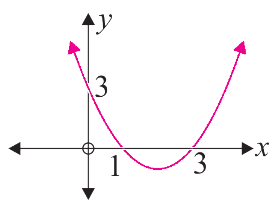
108. Parabola uchini toping:

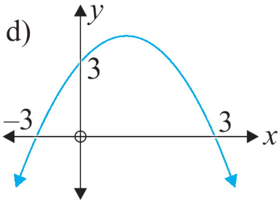
a) $y=x^2-4x+7$;
b) $y=x^2+2x+5$;
c) $f(x)=-x^2+6x-1$;
d) $y=x^2+3$;
e) $f(x)=2x^2+12x$;
f) $y=-3x^2+6x-4$;
g) $y=x^2-x-1$;
h) $y=-2x^2+3x-2$;
i) $y=-\frac{1}{4}x^2+3x-2$.

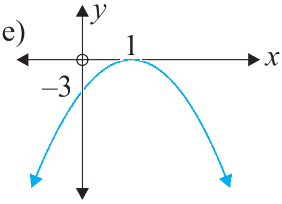
109. Parabolaga qarab, unga mos kvadrat funksiya formulasini toping:

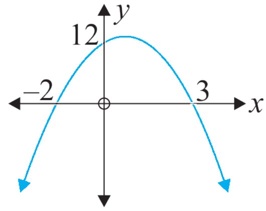
a)


b)


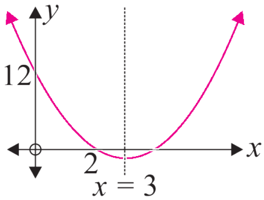
c)


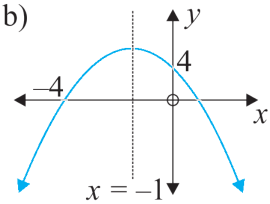
d)


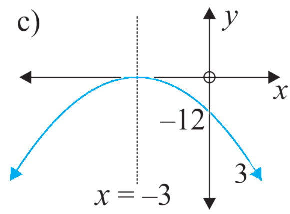
e)


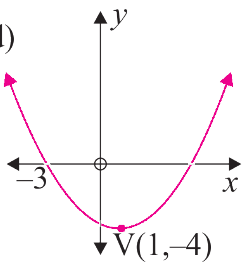
f)


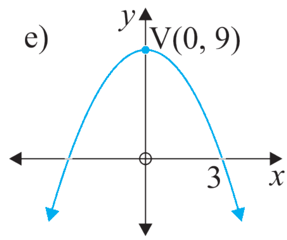
110. Parabolaga qarab, kvadrat funksiyani toping:

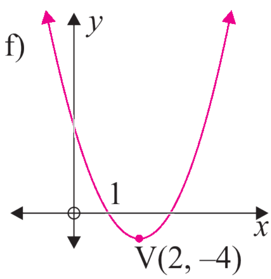
a)


b)


c)


d)


e)


f)


111. Dilshod dengizga durni olish uchun sho'ng'idi. Uning t sekunddan keyingi sho'ng'ish chuqurligi $H(t)=-4t^2+4t+3$ metr bo'ldi, $t \geqslant 0$.

a) durlar qanday chuqurlikda joylashgan?

b) Dilshod durni olish uchun qancha vaqt sarflaydi?

c) Dilshod qanday balandlikdan suvga sho'ng'idi?

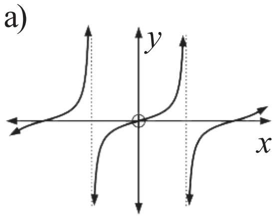
112. Jasmina ko'ylak tikish uchun buyurtma oldi. U bir kunda x ta ko'ylak tiksa, u $P(x)=-x^2+20x$ AQSh dollari miqdorida daromad oladi.

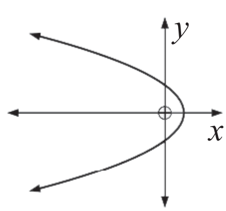
a) eng katta daromad olish uchun u qancha ko'ylak tikish kerak?

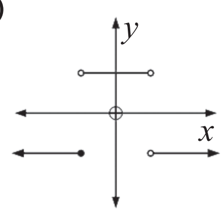
b) Eng katta daromad necha dollarga teng?

Nazorat ishi namunasi

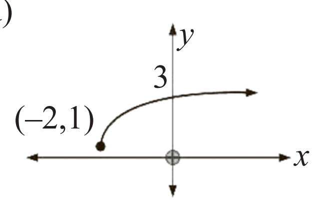
1. Quyidagi munosabatlardan qaysilari funksiyalardir?

a)


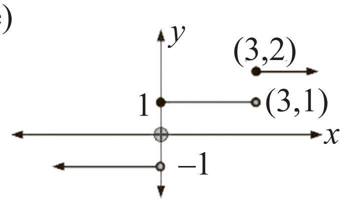
b)


c)


d)



e)

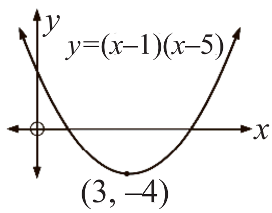


2. Quyidagi tartiblangan juftliklar to‘plamlaridan qaysilari akslantirish bo‘ladi? Javobingizni asoslang.

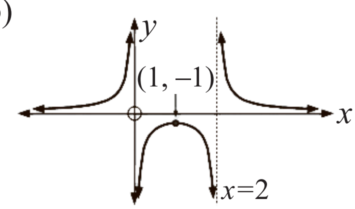
a) $\{(1, 2),(-1, 2),(0, 5),(2, -7)\}$;
b) $\{(0, 1),(1, 3),(2, 5),(0, 7)\}$;
c) $\{(6, 1),(6, 2),(6, 3),(6, 4)\}$.

3. Grafik ko‘rinishda berilgan funksiyalarning aniqlanish sohasini va qiymatlar to‘plamini toping:

a)

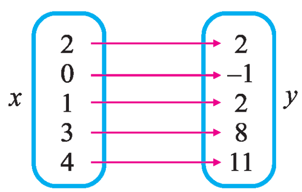


b)

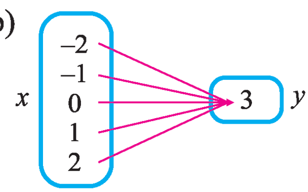


4. Quyidagi diagramma $y=f(x)$ akslantirishni bermoqda.

a)



b)



1) $y=f(x)$ akslantirishning aniqlanish sohasini va qiymatlar to‘plamini yozing.
2) $y=f(x)$ akslantirish tekislikdagi koordinatalar sistemasida qanday tasvirlanadi?
3) $y=f(x)$ uchun aniq ifodani yozing.

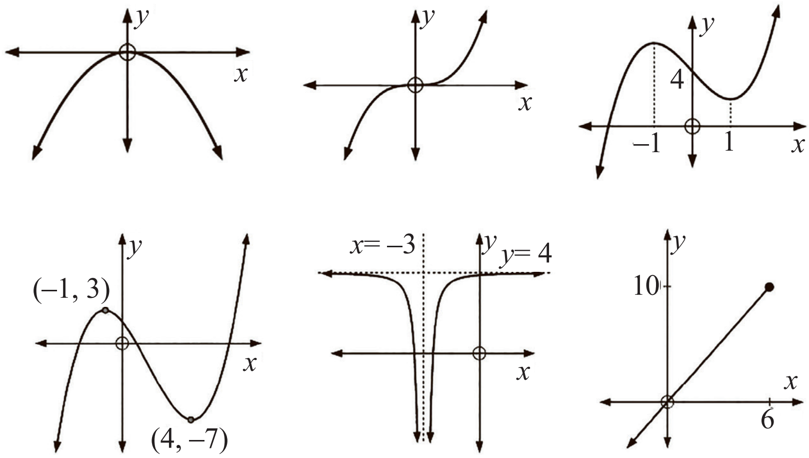
5. $f(x)=2x-x^2$ funksiya uchun:

a) $f(2)$; b) $f(-3)$; c) $f\left(-\frac{1}{2}\right)$ qiymatlarni toping.

6. $g(x)=x^2-3x$ funksiya uchun ifodalarni soddalashtiring:

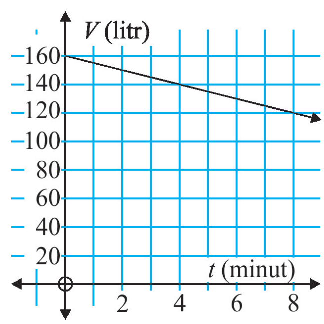
a) $g(x+1)$; b) $g(x^2-2)$.

7. Grafik ko‘rinishda berilgan funksiyalarning kamayish va o‘sish oraliqlarini toping.



8. a) $f(x)=2x+1$; b) $f(x)=-3x+2$; c) $f(x)=x^2$; d) $f(x)=-x^3$ funksiyalar uchun:
1) funksiyalarning o‘qlar bilan kesishish nuqtalarini toping;
2) lokal maksimum, lokal minimum nuqtalari yoki egilish nuqtalari mavjud bo‘lsa, ularning koordinatalarini toping;
3) funksiyalar grafigini taxminiy chizing.

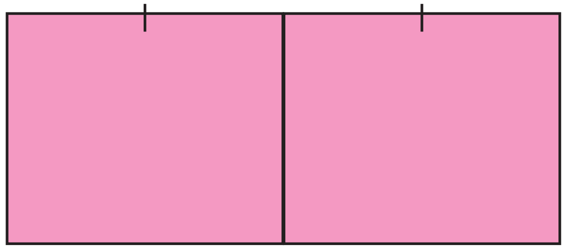
9. Quyidagi grafikda minutlarda ifodalangan t vaqtda sisternadan sizib chiqayotgan neft maxsulotining V hajmi tasvirlangan.



1) Sizib chiqayotgan neft maxsulotining hajmi vaqtga bog‘lanishining formulasini toping.
2) 15 minutda qancha neft sizib chiqadi?
3) 50 litr neft necha minutda sizib chiqadi?
4) Sisterna qancha vaqtdan so‘ng bo‘shaydi?

10. Tosh dengiz sathidan 60 metr balandlikdan yuqoriga uloqtirilgan. t sekunddan so‘ng toshning dengiz sathiga nisbatan balandligi $H(t)=-5t^2+20t+60$ metrga teng bo‘lsa:
1) Necha sekunddan so‘ng toshning balandligi eng katta bo‘ladi?
2) Toshning dengiz sathiga nisbatan balandligi qanchaga teng?
3) Necha sekunddan so‘ng tosh suvga tushadi?

11. Fermer rasmda ko‘rsatilgan ikkita yonma-yon turgan bir xil maydonga ega bo‘lgan bug‘doy dalasini 2000 metr devor bilan o‘radi.



1) Dalalarning umumiy maydoni x orqali qanday ifodalanadi?
2) Ikkita dalaning umumiy maydoni eng ko‘pi bilan kvadtar metrga teng bo‘lishi mumkin? Bunday dalalarning o‘lchamlarini aniqlang.

55 DAVRIY JARAYONLAR VA ULARNI KUZATISH

Davriy jarayonlar tabiatda va texnikada keng tarqalgan. Ularga misollar keltiraylik:

- yil fasllari bo‘yicha ob-havoning o‘zgarishi;
- oylardagi o‘rtacha temperaturaning o‘zgarishi;
- kun va tunning davomiyligi;
- dengiz qirg‘og‘i yonidagi suv chuqurligi;
- hayvonlar soni;
- quyosh faolligining o‘zgarishi;
- Mexanikada, elektrotexnikada davriy tebranishlar.

Bu jarayonlarda muayyan vaqt oraliqlarida takrorlanib turadigan holatlar kuzatiladi. Ular vaziyatga qarab davriy, tebranadigan yoki siklik deyiladi.

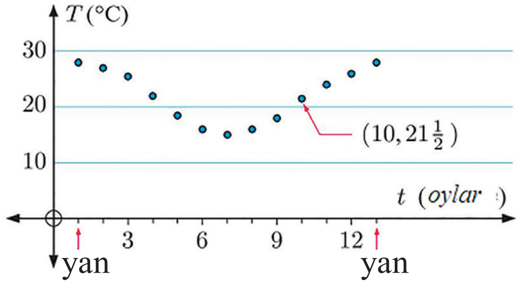
Masalan, Janubiy Afrikadagi Keyptaun shahrida oylik maksimal temperaturaning o‘zgarishini ifodalovchi jadvalni qayarlik:

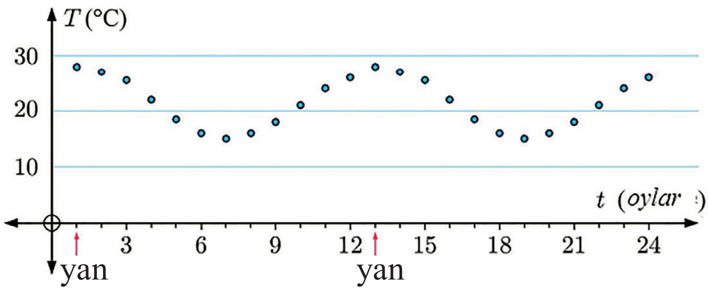
Oy	Yan	Fev	Mar	Apr	May	Iyun	Iyul	Avg	Sen	Okt	Noy	Dek
Temp (0 °C)	28	27	$25\frac{1}{2}$	22	$18\frac{1}{2}$	16	15	16	18	$21\frac{1}{2}$	24	26

Bu ma’lumotlarni grafik ko‘rinishda ifodalaylik. Buning uchun ordinatalar o‘qi temperaturani, abssissalar o‘qi esa oyning tartib raqamini (masalan, fevral uchun $t=2$) bildirsin.

Bu grafikda yanvar oyida o‘rtacha 28 °C temperatura bo‘lishi kuzatilmoqda. Bunday qiymat har yilning yanvarida, ya’ni har 12 oyda takrorlanishi tabiiy.

Boshqa oylar uchun ham o‘rtacha temperatura o‘zgarishini taqribiy aks ettiruvchi grafikni davom ettirsak bo‘ladi:





Agar $y=f(t)$ funksiya t oyda o‘rtacha temperaturani ifodalasa,

$f(0)=f(12)=f(24)=...$, $f(1)=f(13)=f(25)=...$ va h.k. kabi qonuniyat, umumiy holda, ixtiyoriy t uchun $f\ (t+12)$ bo‘lishi kuzatilmoqda.

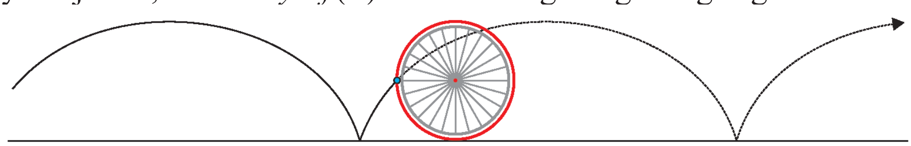
Bunda takrorlanish kuzatiladigan 12 oy muddatni davr deb aytamiz.

X to‘plamda aniqlangan $f(x)$ funksiya uchun ixtiyoriy x da $f(x+T)=f(x)$ tenglikni qanoatlantiradigan $T>0$ mavjud bo‘lsa, $f(x)$ funksiya davriy deyiladi, bunda $x+T \in X$.

Ravshanki, $f(x+T)=f(x)$ bo‘lsa, u holda $f(x)=f(x+T)=f(x+2T)=...$ Bunday $T>0$ sonlarning eng kichik qiymatni funksiyaning davri deb ataymiz.

G‘ildirak to‘g‘ri chiziq bo‘ylab aylanib harakat qilsa, undagi tayin bir belgilangan nuqta *sikloida* deb nomlangan egri chiziq bo‘yicha davriy harakat qiladi.

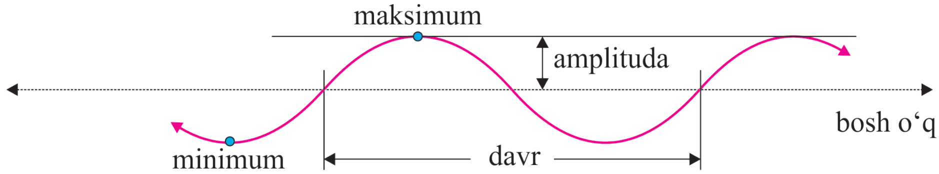
Aytish joizki, sikloida $y=f(x)$ ko‘rinishdagi tenglamaga ega emas.



Davriy funksiyalar grafiklari quyidagi shaklga ega:

Bosh o'q tenglamasi quyidagicha topiladi: $y = \dfrac{\max + \min}{2}$, bunda max – funksiyaning eng katta, min esa eng kichik qiymati.

Davriy funksiya grafigi quyidagi tarkibiy qismlarga ega:



Amplituda funksiyaning maksimumi bilan o'q (yoki o'q bilan minimum) orasidagi masofa bo'lib, u quyidagicha topiladi:

$$\text{amplituda} = \frac{\max - \min}{2}.$$

Savol va topshiriqlar

1. Davriy jarayonga misol keltiring.
2. Funksiyaning davriga ta'rif bering.
3. Davriy funksiyaning amplitudasi qanday hisoblanadi?
4. Sikloidaning nimaligini tushuntiring.
5. Qachon kvadrat funksiya maksimumga (minimumga) ega?

Mashqlar

113. Har bir hol uchun ma'lumotlarni grafik ko'rinishda tasvirlang va ularning davriy–davriymasligi to'g'risida xulosa chiqaring.

a)

x	0	1	2	3	4	5	6	7	8	9	10	11	12
y	0	1	1,4	1	0	–1	–1,4	–1	0	1	1,4	1	0

b)

x	0	1	2	3	4
y	4	1	0	1	4

c)

x	0	0,5	1,0	1,5	2,0	2,5	3,0	3,5
y	0	1,9	3,5	4,5	4,7	4,3	3,4	2,4

d)

x	0	2	3	4	5	6	7	8	9	10	12
y	0	4,7	3,4	1,7	2,1	5,2	8,9	10,9	10,2	8,4	10,4

114. Quyidagi jadvalda g'ildirak to'g'ri chiziq bo'ylab aylanib harakat qilsa, unda belgilangan nuqtaning harakatini ifodalovchi kattaliklar keltirilgan:

Masofa (sm)	0	20	40	60	80	100	120	140	160	180	200
Balandlik (sm)	0	6	23	42	57	64	59	43	23	7	1

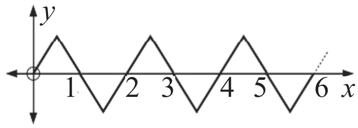
Masofa (sm)	220	240	260	280	300	320	340	360	380	400
Balandlik (sm)	5	27	40	55	63	60	44	24	9	3

a) Balandlikning masofaga bog‘liqligini grafik ko‘rinishda ifodalang.

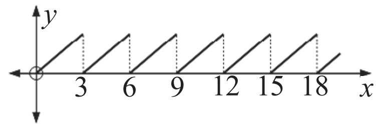
b) Bu jarayon davriymi? Agar davriy bo‘lsa, o‘q tenglamasini, funksiyaning maksimumini, davrini, amplitudasini toping.

115. Quyidagilardan qaysi biri davriy jarayonni ifodalaydi?

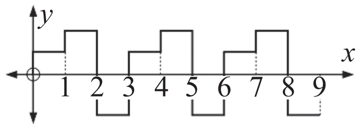
a)



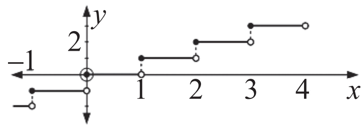
b)



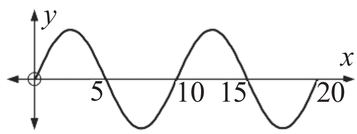
c)



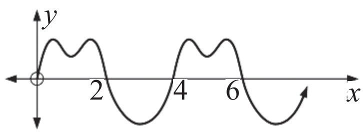
d)



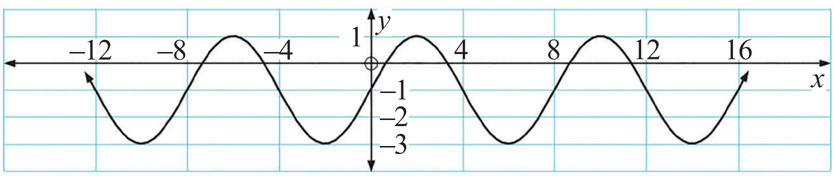
e)



f)



116.



Berilgan davriy funksiya uchun:

a) amplitudani toping; b) davrni toping;

c) birinchi maksimum nuqtasini toping;

d) ikkita maksimum orasidagi masofani aniqlang;

e) bosh o‘qning tenglamasini tuzing.

56-58 $y=\sin x$, $y=\cos x$ FUNKSIYALAR VA ULAR YORDAMIDA MODELLASHTIRISH

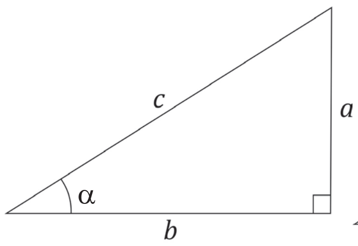
To‘g‘ri burchakli uchburchakda *a, b* – katetlar, *c* – gipotenuza bo‘lsin. α deb *a* katetga qarama-qarshi burchakni belgilaymiz (1- rasmga qarang).

Geometriya kursidan α burchakning sinusi va kosinusi quyidagi tengliklar yordamida kiritiladi:

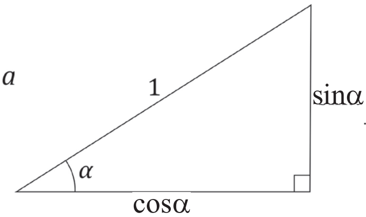
$$\sin\alpha = \frac{a}{c}, \quad \cos\alpha = \frac{b}{c}.$$

Gipotenuzani 1 deb olsak, 1- rasm 2- rasmdagi ko‘rinishni oladi.

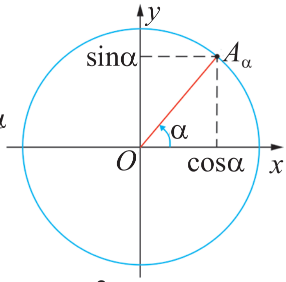
Tekislikda koordinatalar sistemasini kiritib, unda radiusi 1 ga teng aylanani birlik aylanani qaraymiz va shu aylanada α burchakka mos bo‘lgan nuqtani belgilaymiz (3- rasm).



1-rasm.



2-rasm.

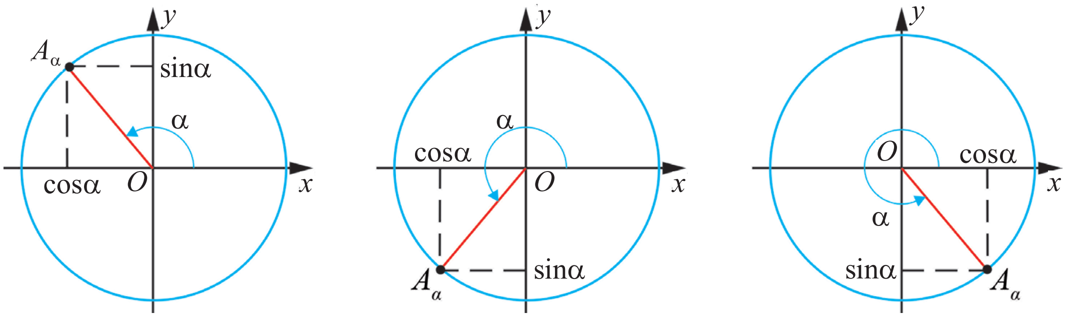


3-rasm.

α burchakning sinusi deb (1;0) nuqtani koordinatalar boshi atrofida α burchakka burish natijasida hosil bo‘lgan A_α nuqtaning ordinatasiga aytiladi (sinα kabi belgilanadi).

Huddi shunday, α burchakning kosinusi deb (1; 0) nuqtani koordinatalar boshi atrofida α burchakka burish natijasida hosil bo‘lgan A_α nuqtaning abssissasiga aytiladi (cosα kabi belgilanadi).

α burchakka mos nuqta boshqa choraklarda yotsa, quyidagi kabi shakllarga ega bo‘lamiz (4- rasm):



4-rasm.

Pifagor teoremasiga ko‘ra, $\cos^2\alpha+\sin^2\alpha=1$ – *asosiy trigonometrik ayniyat o‘rinli, bunda* $0^\circ\leq\alpha\leq360^\circ$. Trigonometriyada qaraladigan burchak (yoy) lar graduslarda yoki radianlarda o‘lchanishi mumkin.

α markaziy burchakka mos yoy uzunligining o‘sha yoy radiusiga nisbati shu burchakning radian o‘lchovi deyiladi.

Graduslarda berilgan α burchakning radian o‘lchovi $\frac{\pi}{180^\circ}\alpha$ ga teng.

Ko‘p uchraydigan burchaklarning radian o‘lchovlari jadvalini keltiramiz:

Gradus	0°	30°	45°	60°	90°	180°	270°	360°
Radian	0	$\frac{\pi}{6}$	$\frac{\pi}{4}$	$\frac{\pi}{3}$	$\frac{\pi}{2}$	π	$\frac{3\pi}{2}$	2π

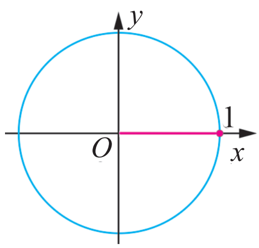
Ayrim α burchaklar sinusi va kosinusi qiymatlarini topaylik.

1. $\alpha = 0°$ bo'lsin (5- rasm). Bu holga mos nuqtaning abssissasi 1 ga, ordinatasi esa 0 ga teng, demak, $\sin 0° = 0$, $\cos 0° = 1$.

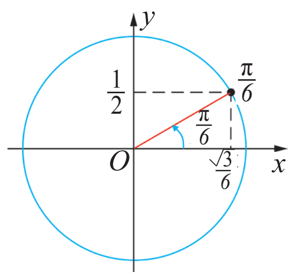
2. $\alpha = \pi/6 = 30°$ bo'lsin (6- rasm). To'g'ri burchakli uchburchakda 30° li burchak qarshisidagi katet gipotenuzaning yarmiga teng bo'lgani bois, $\sin\frac{\pi}{6} = \frac{1}{2}$ bo'ladi. Asosiy trigonometrik ayniyatga ko'ra $\cos\frac{\pi}{6} = \sqrt{1 - \left(\frac{1}{2}\right)^2} = \frac{\sqrt{3}}{2}$.

3. $\alpha = \pi/4 = 45°$ bo'lsin (7- rasm). Bu holda teng yonli to'g'ri burchakli uchburchak hosil bo'ladi.

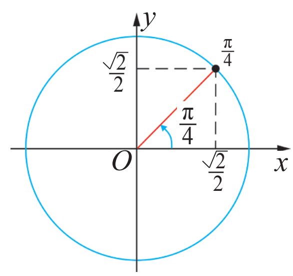
Bunday uchburchakda α burchakning sinusi va kosinusi o'zaro tengdir. Ularni x deylik. Asosiy trigonometrik ayniyatdan $x^2 + x^2 = 1$, ya'ni $x = \frac{\sqrt{2}}{2}$ bo'ladi. Demak, $\cos\frac{\pi}{4} = \sin\frac{\pi}{4} = \frac{\sqrt{2}}{2}$.



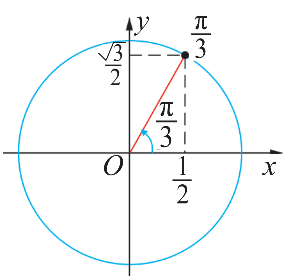
5- rasm.



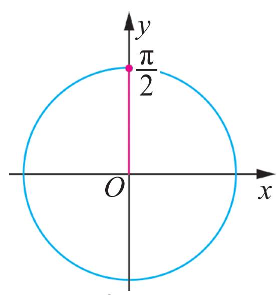
6- rasm.



7- rasm.



8- rasm.

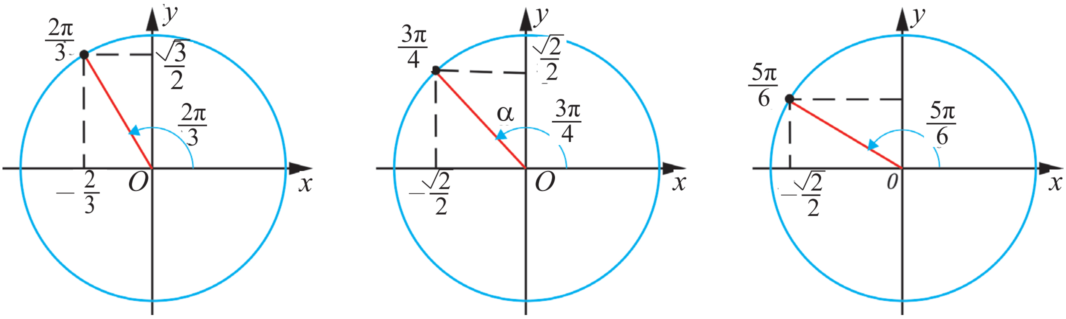


9- rasm.

4. $\alpha = \pi/3 = 60°$ bo'lsin (8- rasm). Bu holda xuddi $\alpha = \frac{\pi}{6}$ holga o'xshash mulohaza yuritib, $\cos\frac{\pi}{3} = \frac{1}{2}$, $\sin\frac{\pi}{3} = \frac{\sqrt{3}}{2}$ tengliklarga ega bo'lamiz.

5. $\alpha = \pi/2 = 90°$ bo'lsin (9- rasm). Bu holga mos nuqtaning abssissasi 0 ga,

ordinatasi esa 1 ga teng. Demak, $\cos\dfrac{\pi}{2}=0$, $\sin\dfrac{\pi}{2}=1$.



10- rasm.

6. $2\pi/3=120°$, $3\pi/4=135°$, $5\pi/6=150°$ bo‘lgan hollarni qaraylik. (10- rasm). $2\pi/3$ nuqta uchun $2\pi/3=\pi-\pi/3$. U holda, bu nuqta $\pi/3$ nuqtaga Oy o‘qiga nisbatan simmetrik. Demak, $\cos\dfrac{2\pi}{3}=-\dfrac{1}{2}$, $\sin\dfrac{2\pi}{3}=\dfrac{\sqrt{3}}{2}$.

$3\pi/4$ nuqta uchun $3\pi/4=\pi-\pi/4$. U holda, bu nuqta $\pi/4$ nuqtaga Oy o‘qiga nisbatan simmetrik. Demak, $\cos\dfrac{3\pi}{4}=-\dfrac{\sqrt{2}}{2}$, $\sin\dfrac{3\pi}{4}=\dfrac{\sqrt{2}}{2}$.

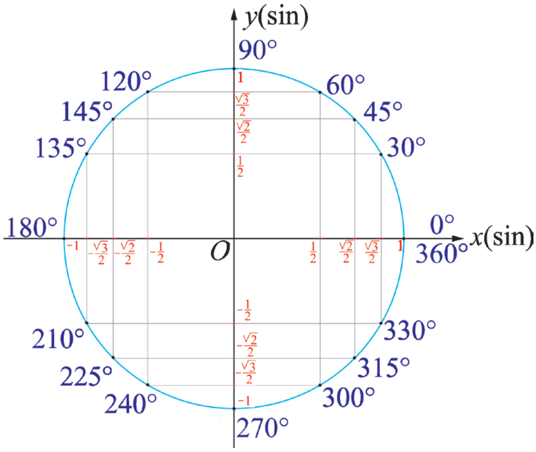
$5\pi/6$ nuqta uchun $5\pi/6=\pi-\pi/6$. U holda, bu nuqta $\pi/6$ nuqtaga Oy o‘qiga nisbatan simmetrik. Demak, $\cos\dfrac{5\pi}{6}=-\dfrac{\sqrt{3}}{2}$, $\sin\dfrac{5\pi}{6}=\dfrac{1}{2}$.

7. $\alpha=\pi=180°$ holda $\cos\pi=-1$, $\sin\pi=0$ ekanini isbotlash va mos rasm chizishni o‘quvchiga havola qilamiz.

Yuqorida biz $[0;\ \pi]$ oraliqda ayrim burchaklar uchun sinus va kosinus qiymatlarini aniqladik. Bu burchaklarning har biriga π ni qo‘shib $[\pi;\ 2\pi]$ oraliqdagi burchaklar uchun ham sinus va kosinus qiymatlarini aniqlash mumkin.

Natijalarni trigonometrik aylana deb nomlangan 11- rasmda ifodalaymiz:

Yuqoridagi qiymatlardan foydalanib $y=\sin x$, $y=\cos x$ funksiyalar grafiklarini yasasa bo‘ladi. Buning uchun abssissalar o‘qida α burchakning qiymatlarini, ordinatalar o‘qida esa sinusning mos qiymatlarini olib, hosil bo‘lgan nuqtalarni belgilaymiz. So‘ng belgilangan nuqtalarni silliq chiziq bilan tutushtirib, $[0;\ 2\pi]$ oraliqdagi $y=\sin x$ (12- rasm) funksiya grafigini hosil qilamiz. $y=\cos x$ (13- rasm) grafigi ham shu kabi yasaladi.



$$30°=\frac{\pi}{6};\quad 45°=\frac{\pi}{4};\quad 60°=\frac{\pi}{6};$$

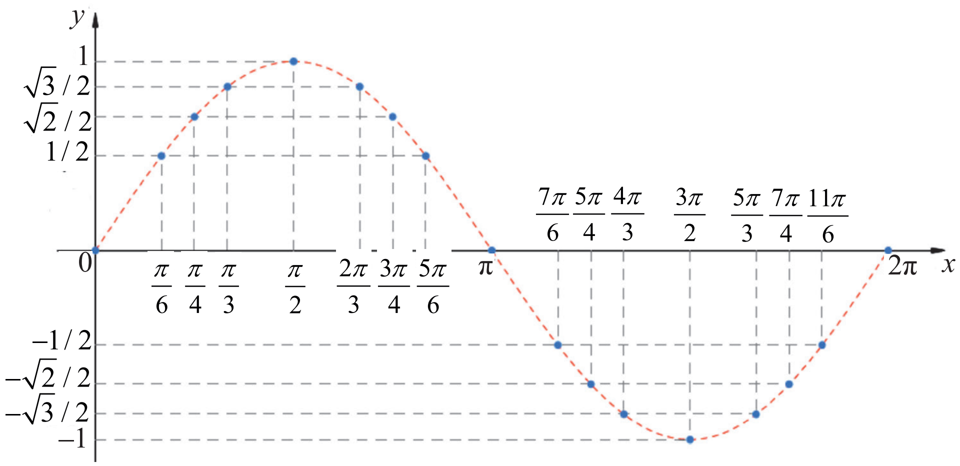
$$90°=\frac{\pi}{2};\quad 120°=\frac{2\pi}{3};\quad 135°=\frac{3\pi}{6};$$

$$180°=\pi;\quad 210°=\frac{7\pi}{6};\quad 225°=\frac{5\pi}{4};$$

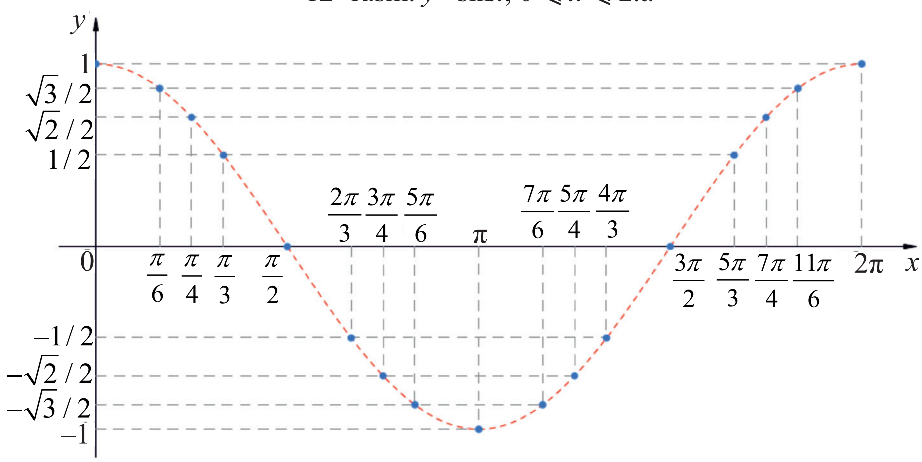
$$240°=\frac{4\pi}{3};\quad 270°=\frac{3\pi}{2};\quad 300°=\frac{5\pi}{3};$$

$$315°=\frac{7\pi}{4};\quad 45°=\frac{11\pi}{6}.$$

11- rasm. Trigonometrik aylana. Sinus va kosinusning ayrim qiymatlari.

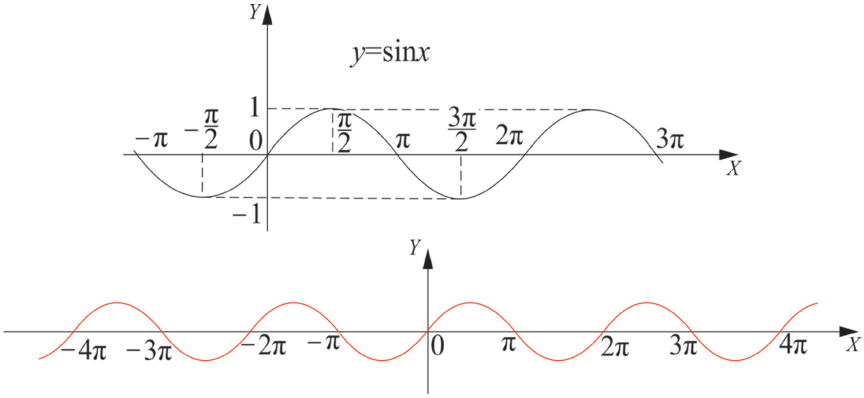


12- rasm. $y=\sin x$, $0 \leqslant x \leqslant 2\pi$.

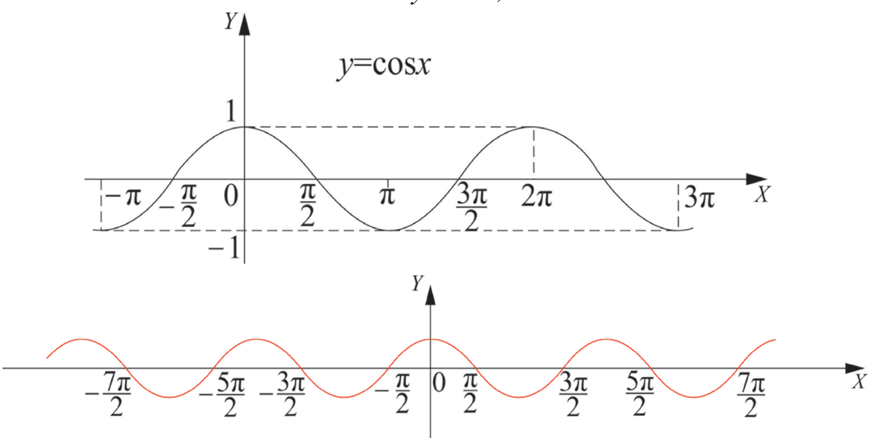


13- rasm. $y=\cos x$, $0 \leqslant x \leqslant 2\pi$.

Bu grafiklarni davriy ravishda davom ettirib, $y=\sin x$, $y=\cos x$ funksiyalarning grafiklarini hosil qilamiz (14 va 15- rasmlar).



14- rasm. $y=\sin x$, $x\in R$.



15- rasm. $y=\cos x$, $x\in R$.

Grafiklarni o‘qib shunday xulosaga kelamiz: $y=\sin x$ ($y=\cos x$) funksiyaning davri 2π ga, amplitudasi 1 ga, eng katta qiymati 1 ga, eng kichik qiymati esa -1 ga teng.

Tatbiqlarda keng uchraydigan $y=a\sin x$ va $y=\sin bx$, $b\neq 0$ funksiyalar to‘g‘risida ba’zi mulohazalarni keltiramiz.

$y=a\sin x$ funksiyaning amplitudasi $|a|$ ga teng. Uning grafigi $y=\sin x$ funksiya grafigini $|a|>1$ bo‘lganda ordinata o‘qi bo‘yicha cho‘zish, $|a|<1$ bo‘lganda esa siqish natijasida hosil bo‘ladi. $y=\sin bx$ funksiyaning davri $\dfrac{360°}{|b|}$ ga teng. Bu funksiyaning grafigi $y=\sin x$ funksiya grafigidan $0<|b|<1$ bo‘lganda abssissa o‘qi bo‘yicha cho‘zish, $|b|>1$ bo‘lganda siqish natijasida hosil bo‘ladi.

$y=\sin x+c$ ko‘rinishdagi funksiya grafigi $y=\sin x$ funksiya grafigini c birlikka parallel ko‘chirish natijasida hosil bo‘ladi va bunda $y=\sin x+c$ funksiyaning bosh o‘qi $y=c$ tenglamaga ega.

Yuqoridagilarni inobatga olib, $y=a\sin bx+c$ ko‘rinishdagi funksiya grafigini hosil qilish mumkin.

Masalan, $y=2\sin 3x+1$ funksiyani qaraylik.

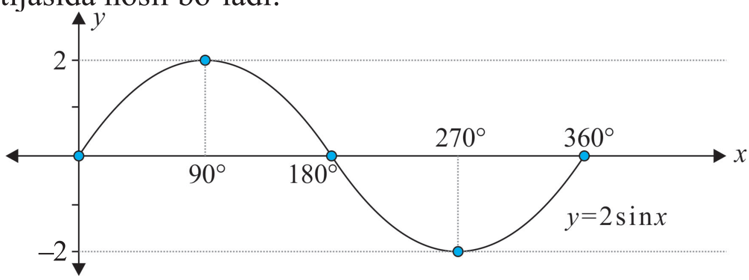
Bu funksiya grafigi $y=\sin x$ funksiya grafigidan quyidagicha hosil bo‘ladi:

1. Amplitudani ikkiga ko‘paytirib $y=2\sin x$ ni hosil qilamiz
2. Davrni uchga bo‘lib, $y=2\sin 3x$ ni hosil qilamiz
3. Berilgan 1 birlikka parallel ko‘chiramiz. $y=2\sin 3x+1$ funksiyaning bosh o‘qi $y=1$ tenglamaga ega.
4. Natijada $y=2\sin 3x+1$ funksiya grafigini hosil qilamiz.

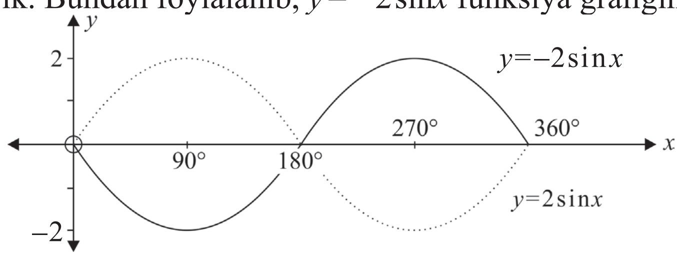
Shunga o‘xshash mulohazalarni $y=\cos x$ funksiya haqida ham keltirsa bo‘ladi.

1- misol. $y=2\sin x$, $y=-2\sin x$, $y=\sin 2x$ funksiyalar grafiklarini yasang, $0°\le x\le 360°$.

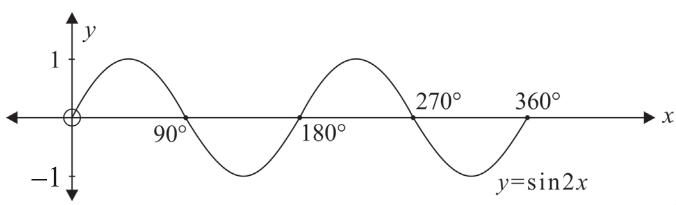
△ Dastlab $y=2\sin x$ funksiya grafigini yasaymiz. Bu funksiyaning amplitudasi 2 ga teng va uning grafigi $y=\sin x$ funksiya grafigini ordinatalar o‘qi bo‘yicha cho‘zish natijasida hosil bo‘ladi:



$y=-2\sin x$ funksiya grafigi $y=2\sin x$ funksiya grafigiga abssissa o‘qiga nisbatan simmetrik. Bundan foylalanib, $y=-2\sin x$ funksiya grafigini yasaymiz.

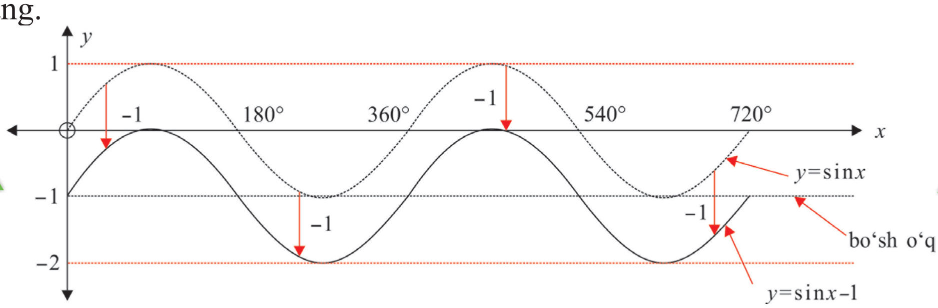


$y=\sin 2x$ funksiyaning davri $\dfrac{360°}{2}=180°$. Bu funksiya grafigi quyidagicha bo‘ladi:



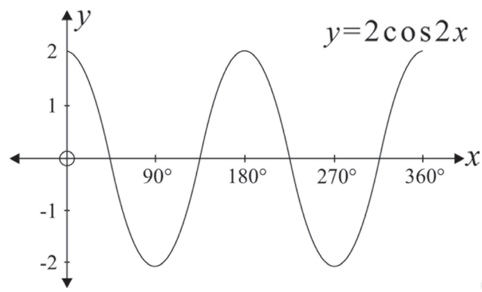
▲

2- misol. $0° \leqslant x \leqslant 720°$ bo'lganda $y=\sin x$ va $y=\sin x-1$ funksiyalar grafiklarini yasang.



3- misol. $0° \leqslant x \leqslant 360°$ kesmada $y=2\cos 2x$ funksiya grafigini yasaylik.

△ $a=2$. Demak, funksiya amplitudasi $|2|=2$ bo'ladi, $b=2$ bo'lgani uchun funksiyaning davri esa $\frac{360°}{|b|}=\frac{360°}{2}=180°$ bo'ladi. Bundan ushbu grafikka ega bo'lamiz:



▲

Savol va topshiriqlar

1. Birlik doirada burchak sinusiga ta'rif bering.
2. Birlik doirada burchak kosinusiga ta'rif bering.
3. 30° li burchak uchun sinus va kosinusni hisoblang.
4. $y=\sin x$ fuksiya grafigini chizing.
5. $y=\cos x$ fuksiya grafigini chizing.

Mashqlar

117. Grafiklarni $0° \leqslant x \leqslant 360°$ kesmada yasang:

a) $y=3\sin x$; b) $y=-3\sin x$; c) $y=\frac{3}{2}\sin x$; d) $y=-\frac{3}{2}\sin x$.

118. Grafiklarni $0° \leqslant x \leqslant 540°$ kesmada yasang:

a) $y=\sin 3x$; b) $y=\sin(\frac{x}{2})$; c) $y=\sin(-2x)$; d) $y=-\sin\frac{x}{3}$.

119. Funksiyaning davrini aniqlang:

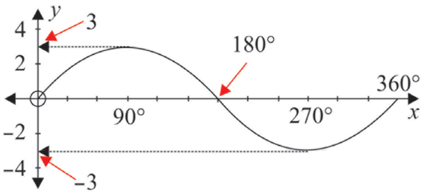
a) $y=\sin 4x$; b) $y=\sin(-4x)$; c) $y=\sin(\frac{x}{3})$; d) $y=\sin(0,6x)$.

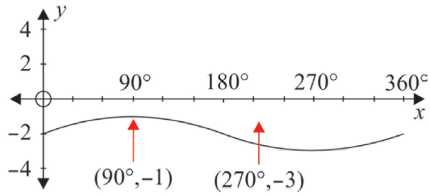
120. Agar $y=\sin bx$, $b>0$ uchun funksiyaning davri

a) 900°; b) 120°; c) 2160°; d) 720°

ga teng bo'lsa, b ni toping.

121. $y=a\sin bx+c$ funksiya grafigiga qarab $a,\ b,\ c$ sonlarni toping:

a)


b)


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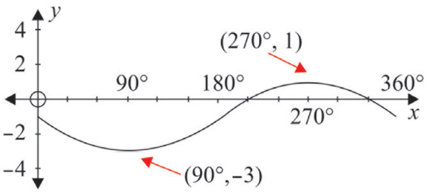

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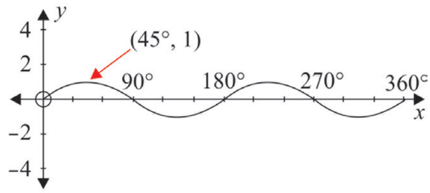

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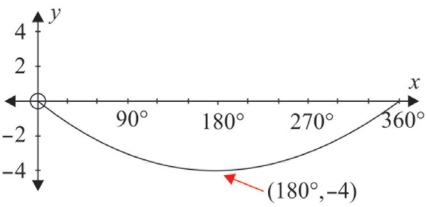
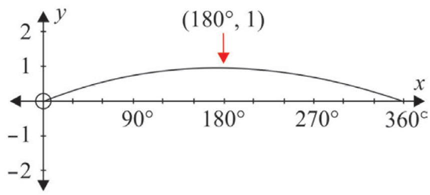


f)


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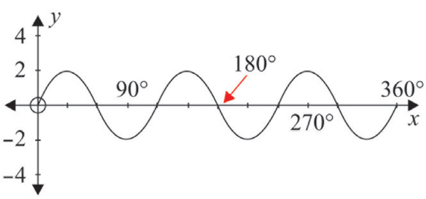

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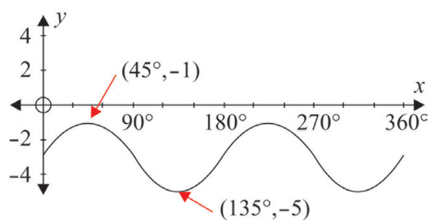
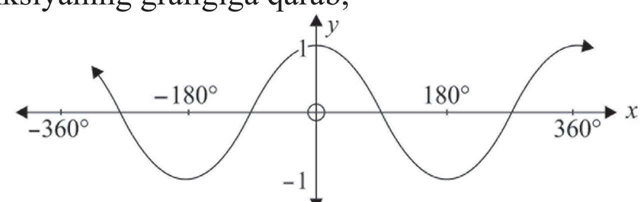


122. Grafiklarni $0°\leqslant x\leqslant 360°$ kesmada yasang:

a) $y=\sin x+1$;　b) $y=\sin x-2$;　c) $y=1-\sin x$;

d) $y=2\sin x-1$;　e) $y=\sin 3x+1$;　f) $y=1-\sin 2x$.

123. $y=\cos x$ funksiyaning grafigiga qarab,



a) $y=\cos x+2$;　b) $y=\cos x-1$;　c) $y=\dfrac{2}{3}\cos x$;

d) $y=\dfrac{3}{2}\cos x$;　e) $y=-\cos x$;　f) $y=\cos 2x$;

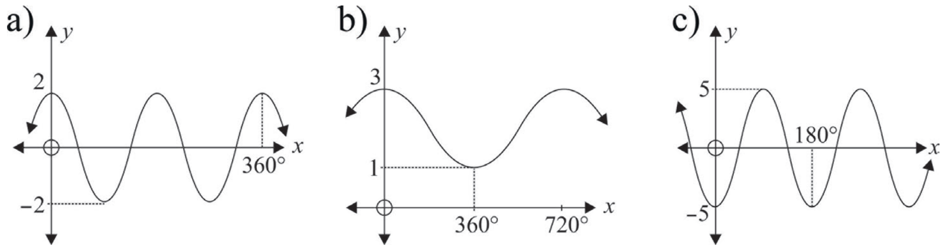
g) $y=\cos(\dfrac{x}{2})$;　h) $y=3\cos 2x$　funksiyalar grafiklarini yasang.

124 Funksiyaning davrini aniqlang:

a) $y=\cos 3x$; b) $y=\cos(\frac{x}{3})$; c) $y=\cos(\frac{x}{2})$; d) $y=\cos 4x$.

125. $y=a\cos bx+c$ funksiya berilgan bo‘lsin. *a, b, c* sonlarning geometrik ma’nosini aniqlang.

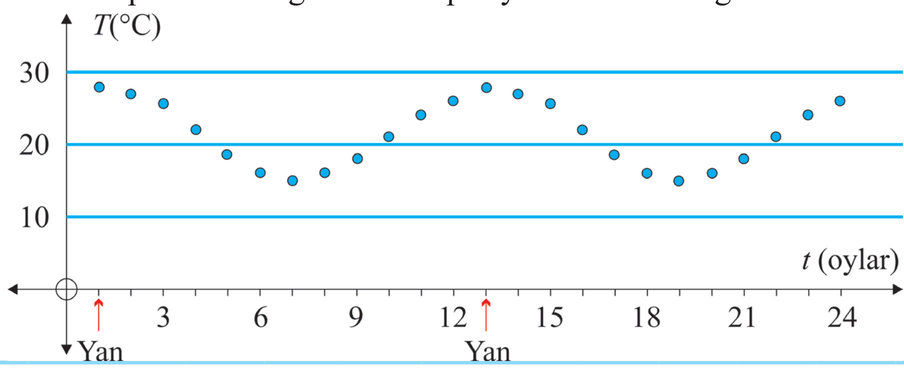
126. $y=a\cos bx+c$ funksiya grafigiga qarab *a, b, c* sonlarni toping.



4- misol. Quyida Janubiy Afrikadagi Keyptaun shahrida ob-havoning oylik maksimal temperaturasining o‘zgarishini ifodalovchi jadval berilgan:

Oy	*Yan*	*Fev*	*Mar*	*Apr*	*May*	*Iyun*	*Iyul*	*Avg*	*Sen*	*Okt*	*Noy*	*Dek*
T(°C)	28	27	$25\frac{1}{2}$	22	$18\frac{1}{2}$	16	15	16	18	$21\frac{1}{2}$	24	26

Maksimal temperatura o‘zgarishini taqribiy aks ettiruvchi grafikni keltiramiz:



Bu jarayonning modeli $T=a\cos bt+c$ ko‘rinishda bo‘lsin deb faraz qilib, parametrlar – *a, b, c* larni topamiz. Davr 12 oy bo‘lgani uchun

$$\frac{360°}{|b|}=12, \text{ ya'ni } b=\frac{360°}{12}=30°.$$

Amplitudani hisoblaymiz: $\frac{\max-\min}{2}\approx\frac{28-15}{2}=6,5$. Bundan α≈6,5.

Bosh o‘q maksimal va minimal qiymatlar to‘g‘ri chiziqlari o‘rtasida bo‘lgani bois $c\approx\frac{28+15}{2}\approx 21,5$.

Demak, maksimal oylik temperatura vaqt oʻtishi bilan oʻzgarishining matematik modeli $T \approx 6{,}5 \cos 30t + 21{,}5$ funksiyadir.

Mashqlar

127. Antarktidadagi Qutb bazasida 30 yil mobaynida oʻrtacha temperatura quyidagicha boʻlganligi maʼlum:

Oyning tartib raqami	1	2	3	4	5	6	7	8	9	10	11	12
Temperatura (°C)	0	−4	−10	−15	−16	−17	−18	−19	−17	−13	−6	−1

Oʻrtacha temperatura oʻzgarishining matematik modelini tuzing.

128. Dengiz qirgʻogʻida dengiz suvining koʻtarilishi va orqaga qaytishi jarayoni kuzatilganda quyidagilar aniqlandi: 1) suv chuqurligining eng katta va eng kichik qiymatlari orasidagi farq 14 metr; 2) suv chuqurligi eng katta qiymatlarga oʻrtacha har 12,4 soatda erishadi. Suv chuqurligining vaqtga nisbatan oʻzgarishining matematik modelini tuzing va uni grafik koʻrinishda ifodalang.

129. Velosiped gʻildiragida sariq rangli nur qaytargʻich oʻrnatilgan. Velosiped kechasi tekis yoʻl boʻylab harakatlanganda u videotasvirga olindi. Videotasvir asosida nur qaytargʻichning yoʻlga nisbatan balandligi vaqt oʻtishi bilan qanday oʻzgargani aniqlanib, quyidagi jadval toʻldirildi:

Vaqt (t)	0,1	0,2	0,3	0,4	0,5	0,6	0,7	0,8	0,9
Balandlik (H, sm)	19	17	38	62	68	50	24	15	31

a) Sinus funksiyasidan foydalanib, jarayonning matematik modelini tuzing;

b) jarayonning grafik koʻrinishini keltiring;

c) gʻildirakning radiusni toping;

d) velosiped qanday tezlikda harakatlanmoqda?

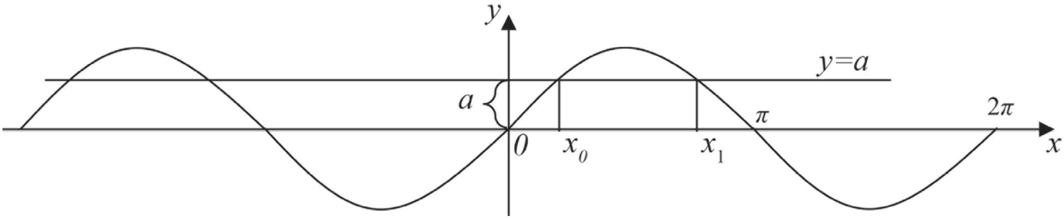
59-61 ENG SODDA TRIGONOMETRIK TENGLAMALAR

$\sin x = a$ tenglama

Bizga maʼlumki, $-1 \le \sin x \le 1$, shuning uchun bu tenglama $|a|>1$ boʻlganida yechimga ega emas. $-1 \le a \le 1$ oraliqda tenglamaning yechimini topish uchun quyidagi taʼrifni kiritamiz.

$a\in[-1;\ 1]$ sonning *arksinusi* deb sinusi a ga teng bo‘lgan $x\in[-\frac{\pi}{2};\frac{\pi}{2}]$ songa aytiladi: agar $\sin x=a$ va $x\in[-\frac{\pi}{2};\frac{\pi}{2}]$ bo‘lsa, $\arcsin a=x$.

Tenglamani yechish uchun 16- rasmdagi $y=\sin x$ funksiya grafigidan foydalanamiz.



16- rasm.

Grafikdan ko‘rinadiki, $a\in[-1;\ 1]$ bo‘lganda $y=a$ funksiya $[0;\ 2\pi]$ oraliqda $y=\sin x$ funksiya grafigini abssissalari x_0 va $x_1=\pi-x_0$ bo‘lgan nuqtalarda kesadi. Bu ikki nuqtani bitta formula orqali yozish mumkin:

$$x=(-1)^n \arcsin a,\ n=0,\ 1.$$

$y=\sin x$ funksiyaning davriyligidan foydalanib, tenglamani yechish uchun ushbu formulani hosil qilamiz:

$$x=(-1)^k \arcsin a+\pi k,\ k\in Z. \qquad (1)$$

1- misol. Hisoblang: 1) $\arcsin\frac{\sqrt{3}}{2}$; 2) $\arcsin\left(-\frac{1}{2}\right)$.

△ Ta’rifga ko‘ra $-1\le\frac{\sqrt{3}}{2}\le 1$, $\frac{\pi}{3}\in[-\frac{\pi}{2};\frac{\pi}{2}]$ va $\sin\frac{\pi}{3}=\frac{\sqrt{3}}{2}$ bo‘lgani uchun $\arcsin\frac{\sqrt{3}}{2}=\frac{\pi}{3}$. Huddi shuningdek, $\arcsin(-\frac{1}{2})=-\arcsin\frac{1}{2}=-\frac{\pi}{6}$ bo‘ladi. ▲

2- misol. Tenglamani yeching: $\sin x=\frac{1}{2}$.

△ (1) Formulaga ko‘ra tenglamaning yechimi

$x=(-1)^k\arcsin\frac{1}{2}+\pi k=(-1)^k\frac{\pi}{6}+\pi k,\ \ k\in Z$ bo‘ladi. ▲

3-misol. Tenglamani yeching: $\sin\left(\frac{\pi}{12}-\frac{x}{2}\right)=\frac{\sqrt{2}}{2}$.

△ $y=\sin x$ funksiya toq bo‘lgani uchun $\sin\left(\frac{x}{2}-\frac{\pi}{12}\right)=-\frac{\sqrt{2}}{2}$ bo‘ladi.

(1) formulani qo‘llab, $\frac{x}{2}-\frac{\pi}{12}=(-1)^k\arcsin\left(-\frac{\sqrt{2}}{2}\right)+\pi k,\ \ k\in Z$ tenglikni ho-

sil qilamiz. $\arcsin\left(-\frac{\sqrt{2}}{2}\right)=-\frac{\pi}{4}$ bo'lgani uchun $\frac{x}{2}-\frac{\pi}{12}=(-1)^k\left(-\frac{\pi}{4}\right)+\pi k$, $\frac{x}{2}-\frac{\pi}{12}=(-1)^k\left(-\frac{\pi}{4}\right)+\pi k$, yoki $x=\frac{\pi}{6}+(-1)^{k+1}\frac{\pi}{2}+2k\pi,\ k\in Z$ yechimlarni olamiz. ▲

$\sin x=a$ tenglamaning muhim hollardagi yechimlarini keltiramiz:

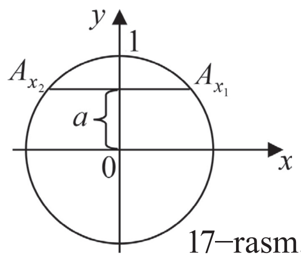
$a=1$ bo'lganda $x=\frac{\pi}{2}+2\pi k,\ k\in Z$; $a=-1$ bo'lganda $x=\frac{3}{2}\pi+2\pi k,\ k\in Z$;

$a=0$ bo'lganda $x=\pi k,\ k\in Z$.

4-misol. Tenglamani yeching: $\sin\left(\frac{\pi}{10}-\frac{x}{2}\right)=0$.

△ $a=0$ ekanidan $-\frac{\pi}{10}+\frac{x}{2}=\pi k$, $\frac{x}{2}=\pi k+\frac{\pi}{10}$, ya'ni $x=\frac{\pi}{5}+2\pi k,\ k\in Z$ yechimlarni topamiz. ▲

$\sin x=a$ tenglamani yechishni birlik doirada tushuntirish oson. $\sin x$ ning ta'rifiga ko'ra, uning qiymati birlik doiradagi A_x nuqtaning ordinatasidir. $|a|<1$ bo'lganda bunday nuqtalar 2 ta, ya'ni A_{x_1} va A_{x_2}. $a=\pm1$ bo'lganda esa 1 ta (17- rasm).



17–rasm.

$\cos x=a$ tenglama

$-1\le\cos x\le1$ bo'lgani uchun bu tenglama $|a|>1$ bo'lganda yechimga ega emas. $-1\le a\le1$ oraliqda tenglama yechimini topish uchun quyidagi ta'rifni kiritamiz.

$a\in[-1;\ 1]$ sonning ***arkkosinusi*** deb kosinusi a ga teng bo'lgan $x\in[0;\ \pi]$ songa aytiladi: agar $\cos x=a$ va $x\in[0;\ \pi]$ bo'lsa, $\arccos a=x$.

Ta'rifga ko'ra, $[0;\ \pi]$ oraliqda $\cos x=a$ tenglama bitta $x=\arccos a$ ildizga ega. $y=\cos x$ funksiya juft bo'lganligi uchun $[-\pi;0]$ oraliqda ham bitta $x=-\arccos a$ yechimga ega. Funksiyaning davri 2π. U holda $\cos x=a$ tenglamani yechish uchun $x=\pm\arccos a+2\pi k,\ k\in Z$ (2) formulani hosil qilamiz.

5- misol. Hisoblang: 1) $\arccos\frac{\sqrt{3}}{2}$; 2) $\arccos\left(-\frac{\sqrt{2}}{2}\right)$.

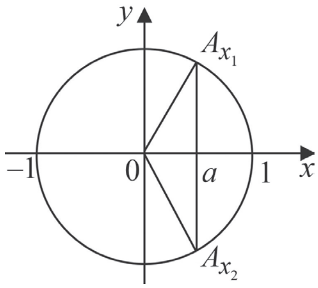
△ Ta'rifga ko'ra, $-1\le\frac{\sqrt{3}}{2}\le1$, $\frac{\pi}{6}\in[0;\pi]$ va $\cos\frac{\pi}{6}=\frac{\sqrt{3}}{2}$ bo'lgani uchun $\arccos\frac{\sqrt{3}}{2}=\frac{\pi}{6}$ bo'ladi. Huddi shuningdek, $\arccos\left(-\frac{\sqrt{2}}{2}\right)=\frac{3\pi}{4}$ bo'ladi. ▲

6-misol. Tenglamani yeching: $\cos x = \frac{\sqrt{3}}{2}$.

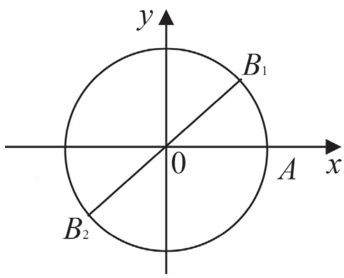
△(2) formulaga ko'ra tenglamaning yechimini $x = \pm\arccos\frac{\sqrt{3}}{2} + 2\pi k,\ k \in Z$

ammo $\arccos\frac{\sqrt{3}}{2} = \frac{\pi}{6}$.

Demak, yechim ushbu ko'rinishda bo'ladi: $x = \pm\frac{\pi}{6} + 2\pi k,\ k \in Z$ ▲



18-rasm.



19-rasm.

$\cos x = a$ tenglama yechilishini birlik doirada tushuntiramiz (18- rasm). $\cos x$ funksiyaning ta'rifiga ko'ra uning qiymati birlik doiradagi A_x nuqtaning abssissasi bo'ladi. $|a|<1$ bo'lganda bunday nuqtalar 2ta, ya'ni A_{x_1} va A_{x_2}; $a=1$ va $a=-1$ bo'lganda bunday nuqta bitta.

$\cos x = a$ tenglamaning muhim hollardagi yechimlarini keltiramiz:

$a=1$ bo'lganda $x=2\pi k,\ k\in Z;$ $a=-1$ bo'lganda $x=\pi+2\pi k,\ k\in Z;$

$a=0$ bo'lganda $x = \frac{\pi}{2} + \pi k,\ k\in Z.$

7- misol. Tenglamani yeching: $\cos\left(3x - \frac{\pi}{4}\right) = 0$.

△$\cos x=0$ tenglamaning yechimi formulasidan $3x - \frac{\pi}{4} = \frac{\pi}{2} + \pi k$ ni hosil qilamiz. Bundan, $x = \frac{\pi}{4} + \frac{\pi k}{3},\ k \in Z$.▲

tg$x=a$ tenglama

Bu tenglamani yechish uchun quyidagi ta'rifni kiritamiz. $a\in R$ sonning *arktangensi* deb, tangensi a songa teng bo'lgan $x\in(-\pi/2;\pi/2)$ songa aytiladi: agar $\mathrm{tg}x=a$ va $x\in(-\pi/2;\pi/2)$ bo'lsa, $\mathrm{arctg}\, a=x$.

$\mathrm{tg}x = \frac{\sin x}{\cos x}$ bo'lgani uchun tg$x$ birlik doiradagi $B\ (x;\ y)$ nuqta ordinatasining abssissasiga nisbatiga teng (19- rasm), ya'ni bu nuqta $\frac{y}{x} = a$ to'g'ri chiziq bilan birlik doiraning kesishish nuqtasidir. 19- rasmga ko'ra bunday nuqtalar 2 ta: B_1 va B_2 nuqtalar. Shuning uchun tenglamaning yechimi quyidagicha bo'ladi:

$$x = \mathrm{arctg}\, a + \pi n,\ n \in Z. \qquad (3)$$

8- misol. Hisoblang: 1) arctg1; 2) $\mathrm{arctg}(-\sqrt{3})$.

△ 1) $\mathrm{tg}\frac{\pi}{4} = 1$ va $\frac{\pi}{4} \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ bo'lgani uchun $\mathrm{arctg}\,1 = \frac{\pi}{4}$;

2) $\mathrm{tg}\left(-\frac{\pi}{3}\right) = -\sqrt{3}$ va $-\frac{\pi}{3} \in \left(-\frac{\pi}{2}; \frac{\pi}{2}\right)$ bo'lgani uchun $\mathrm{arctg}\left(-\sqrt{3}\right) = -\frac{\pi}{3}$. ▲

9- misol. Tenglamani yeching: $\mathrm{tg}\left(x - \frac{\pi}{6}\right) = -\sqrt{3}$.

△ (3)ga ko'ra, tenglamaning yechimlari quyidagicha bo'ladi:

$x - \frac{\pi}{6} = \mathrm{arctg}(-\sqrt{3}) + \pi n$. $\mathrm{arctg}(-\sqrt{3}) = -\mathrm{arctg}(\sqrt{3}) = -\frac{\pi}{3}$ bo'lgani uchun tenglamaning yechimlari $x - \frac{\pi}{6} = -\frac{\pi}{3} + \pi n$, yoki $x = -\frac{\pi}{6} + \pi n,\ n \in Z$. ▲

Eng sodda trigonometrik tenglamalar uchun jadvalni keltramiz:

Tenglama	Yechimlari	Ba'zi xossalar
$\sin x = a$	$x = (-1)^k \arcsin a + \pi k,\ k \in Z.$	$\arcsin(-a) = -\arcsin a,\ \lvert a \rvert \leqslant 1.$
$\cos x = a$	$x = \pm \arccos a + 2\pi k,\ k \in Z.$	$\arccos(-a) = \pi - \arccos a,\ \lvert a \rvert \leqslant 1.$
$\mathrm{tg}\, x = a$	$x = \mathrm{arctg}\, a + \pi k,\ k \in Z.$	$\mathrm{arctg}(-a) = -\mathrm{arctg}\, a,\ a \in R.$

Uchinchi ustunda keltirilgan xossalar manfiy sonlar arksinuslari (arkkosinuslari, arktangenuslari) qiymatlarini musbat sonlar arksinuslari qiymatlari orqali topish imkoniyatini beradi. Masalan, $\arcsin\left(-\frac{\sqrt{2}}{2}\right) = -\arcsin\frac{\sqrt{2}}{2} = -\frac{\pi}{4}$,

$$\arccos\left(-\frac{\sqrt{3}}{2}\right) = \pi - \arccos\frac{\sqrt{3}}{2} = \pi - \frac{\pi}{6} = \frac{5\pi}{6},$$

$$\mathrm{arctg}\left(-\frac{\sqrt{3}}{3}\right) = -\mathrm{arctg}\frac{\sqrt{3}}{3} = -\frac{\pi}{6}.$$

10- misol. Tenglamani yeching: $\cos(10x + \frac{\pi}{8}) = \frac{1}{2}$.

△ $10x + \frac{\pi}{8} = z$ belgilash kiritib, $\cos z = \frac{1}{2}$ tenglamani hosil qilamiz. Bundan (2) formulaga ko'ra $z = \pm\frac{\pi}{3} + 2\pi k,\ k \in Z$, ya'ni $10x + \frac{\pi}{8} = \pm\frac{\pi}{3} + 2\pi k$ yoki

$$x=\frac{1}{10}\left(-\frac{\pi}{8}\pm\frac{\pi}{3}+2\pi k\right),\ k\in Z.$$ ▲

sin$x$=sin$a$, cos$x$=cos$b$, tg$x$=tg$c$ **ko'rinishdagi tenglamalar**

Bunday tenglamalarning yechimi, mos ravishda, quyidagicha bo'ladi:
$x=(-1)^k a+\pi k;\ k\in Z;\quad x=\pm b+2\pi n,\ n\in Z;\quad x=c+\pi m,\ m\in Z.$ (4)

11- misol. Tenglamani yeching: $\cos(3x-40°)=\cos(2x+60°)$.

△(4) formulaga ko'ra, $3x-40°=\pm(2x+60°)+360°n,\ n\in Z$ tenglamani hosil qilamiz. Bundan no'malum x topiladi:

$$3x-40°=2x+60°+360°n \Leftrightarrow x=100°+360°n,\ n\in Z;$$
$$3x-40°=-2x-60°+360°n,\ 5x=-20°+360°n \Leftrightarrow x=-4°+72°n,\ n\in Z.$$ ▲

12- misol. Tenglamani yeching: $\sin^2 x+3\sin x+2=0$.

△$\sin x=z$ belgilash kiritib, $z^2+3z+2=0$ kvadrat tenglamaga kelamiz. Bu tenglamani yechib $z_1=-2,\ z_2=-1$ lar topiladi. Belgilashga ko'ra $\sin z=-2$ va $\sin x=-1$ tenglamalarni hosil qilamiz. $\sin z=-2$ yechimga ega emas. $\sin x=-1$ tenglama $x=270°+360°k,\ k\in Z$ yechimga ega. Demak, tenglamaning yechimi $x=270°+360°k,\ k\in Z$ bo'ladi. ▲

Savol va topshiriqlar

1. sin$x$=a tenglama qanday yechiladi? Misolda tushuntiring.
2. cos$x$=a tenglama qanday yechiladi? Misol keltiring.
3. tg$x$=a tenglama qanday yechiladi? Misol yordamida tushuntiring.
4. arcsin a soniga ta'rif bering. Misolda tushuntiring.
5. arccos a soniga ta'rif bering. Misolda tushuntiring.
6. arctg a soniga ta'rif bering. Misolda tushuntiring.

Mashqlar

Hisoblang (**130–141**):

130. 1) arcsin 0; 2) arcsin $\frac{\sqrt{3}}{2}$; 3) arcsin $\frac{1}{2}$; 4) arcsin $\left(-\frac{\sqrt{3}}{2}\right)$.

131. 1) arcsin $\left(-\frac{\sqrt{2}}{2}\right)$; 2) arcsin $\left(-\frac{1}{2}\right)$; 3) arcsin 1; 4) arcsin (-1).

132. 1) arccos 0; 2) arccos $\left(-\frac{\sqrt{3}}{2}\right)$; 3) arccos $\frac{\sqrt{2}}{2}$; 4) arccos (-1).

133. 1) arccos $\left(-\frac{1}{2}\right)$; 2) arccos $\frac{1}{2}$; 3) arccos 1; 4) arccos $\left(-\frac{\sqrt{2}}{2}\right)$.

134. 1) $\operatorname{arctg} 1$; | 2) $\operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right)$; | 3) $\operatorname{arctg}\frac{1}{\sqrt{3}}$; | 4) $3\cdot\operatorname{arctg}\left(-\sqrt{3}\right)$.

135. 1) $\operatorname{arctg} 0$; | 2) $\operatorname{arctg}\left(-\sqrt{3}\right)$; | 3) $\operatorname{arctg}(-1)$; | 4) $7\cdot\operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right)$.

136. 1) $\arcsin 1+\arcsin(-1)$; | 2) $2\arcsin\frac{\sqrt{3}}{2}+4\arcsin\frac{1}{2}$.

137. 1) $4\arcsin\frac{\sqrt{2}}{2}-2\arcsin\left(-\frac{\sqrt{2}}{2}\right)$; | 2) $\arcsin\left(-\frac{1}{2}\right)+\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

138. 1) $2\arccos 1+3\arccos 0$; | 2) $6\arccos\frac{\sqrt{3}}{2}-3\arccos\left(-\frac{1}{2}\right)$.

139. 1) $2\arccos(-1)-3\arccos 0$; | 2) $2\arccos\left(-\frac{\sqrt{2}}{2}\right)+4\arccos\left(-\frac{\sqrt{3}}{2}\right)$.

140. 1) $3\operatorname{arctg}\sqrt{3}+3\arccos\frac{1}{2}$; | 2) $3\operatorname{arctg}\left(-\frac{1}{\sqrt{3}}\right)+2\arccos\left(-\frac{\sqrt{3}}{2}\right)$.

141. 1) $2\operatorname{arctg} 1+3\arcsin\left(-\frac{1}{2}\right)$; | 2) $5\operatorname{arctg}(-\sqrt{3})-3\arccos\left(-\frac{\sqrt{2}}{2}\right)$.

Ifodalar ma’noga ega yoki ega emasligini aniqlang (**142–143**):

142. 1) $\arccos(\sqrt{8}-3)$; | 2) $\arcsin(2-\sqrt{15})$; | 3) $\arccos(3-\sqrt{18})$.

143. 1) $\operatorname{tg}\left(2\arcsin\frac{\sqrt{2}}{2}\right)$; | 2) $\arcsin(\sqrt{6}-2)$; | 3) $\operatorname{tg}\left(3\arccos\frac{1}{2}\right)$.

Tenglamani yeching (**144–161**):

144. 1) $\sin x=-\frac{1}{2}$; | 2) $\sin x=\frac{\sqrt{3}}{2}$; | 3) $\sin x=-\frac{\sqrt{2}}{2}$; | 4) $\sin 2x=\frac{1}{2}$.

145. 1) $\sin x=\frac{\sqrt{2}}{2}$; | 2) $\sin x=1$; | 3) $\sin x=-\frac{\sqrt{3}}{2}$; | 4) $\sin 2x=\frac{\sqrt{3}}{2}$.

146. 1) $\cos x=-\frac{\sqrt{2}}{2}$; | 2) $\cos x=\frac{\sqrt{3}}{2}$; | 3) $\cos 2x=-1$; | 4) $\cos 3x=1$.

147. 1) $\cos x=\frac{1}{2}$; | 2) $\cos x=-1$; | 3) $\cos 5x=-\frac{1}{2}$; | 4) $\cos 3x=-1$.

148. 1) $\operatorname{tg}x=-\sqrt{3}$; | 2) $\operatorname{tg}x=1$; | 3) $\operatorname{tg}9x=-1$; | 4) $\operatorname{tg}3x=\frac{\sqrt{3}}{3}$.

149. 1) $\operatorname{tg}x=0$; | 2) $\operatorname{tg}x=2$; | 3) $\operatorname{tg}6x=-3$; | 4) $\operatorname{tg}5x=-\frac{\sqrt{3}}{3}$.

150. 1) $2\cos x+1=0$; | 2) $2\cos x-\sqrt{3}=0$; | 3) $2\cos x-\sqrt{2}=0$.

151. 1) $\sqrt{2}\sin x-1=0$; | 2) $2\sin x+\sqrt{3}=0$; | 3) $2\sin x+\sqrt{2}=0$.

152. 1) $\sin\left(-\frac{x}{2}\right)=\frac{\sqrt{3}}{2}$; | 2) $\operatorname{tg}4x=-\frac{1}{\sqrt{3}}$; | 3) $\cos(-3x)=\frac{\sqrt{2}}{2}$.

153. 1) $2\sin\left(2x+\frac{\pi}{4}\right)=-\sqrt{2}$; | 2) $\sqrt{3}\operatorname{tg}\left(\frac{x}{2}-\frac{\pi}{3}\right)=1$; | 3) $2\cos(\frac{x}{3}-\frac{\pi}{6})=\sqrt{3}$.

154. 1) $\cos\left(\frac{\pi}{3}-2x\right)=-1$; | 2) $\operatorname{tg}\left(\frac{\pi}{4}+\frac{x}{3}\right)=1$; | 3) $2\cos\left(\frac{\pi}{6}-\frac{x}{2}\right)=\sqrt{3}$.

155. 1) $2\sin\left(\frac{\pi}{6}-\frac{x}{2}\right)=\sqrt{3}$; | 2) $2\cos\left(\frac{\pi}{4}-3x\right)=\sqrt{2}$; | 3) $\sin\left(x+\frac{\pi}{4}\right)=1$.

156. 1) $(2\sin x+\sqrt{2})(\sin 4x+1)=0$; | 2) $(2-\cos x)(1+3\cos x)=0$.

157. 1) $2\sin^2 x-\sin x-1=0$; | 2) $4\cos^2 x-8\cos x-3=0$;
3) $2\sin^2 x-\sin x-6=0$; | 4) $2\cos^2 x-\cos x-6=0$.

158. 1) $2\sin^2 x-\sin x-1=0$; | 2) $4\cos^2 x-8\cos x-3=0$;
3) $2\sin^2 x-\sin x-6=0$; | 4) $2\cos^2 x-\cos x-6=0$.

159. 1) $2\cos^2 x-\sin x+1=0$; | 2) $\operatorname{tg}^2 x-3\operatorname{tg}x-4=0$;
3) $4\sin^2 x-\cos x-1=0$; | 4) $\operatorname{tg}x-\sqrt{3}\operatorname{tg}x+1=\sqrt{3}$.

160. 1) $\cos x=\cos 2x$; | 2) $\operatorname{tg}2x=\operatorname{tg}3x$; | 3) $\sin 7x=\sin 3x$; | 4) $\cos 4x=\cos 5x$.

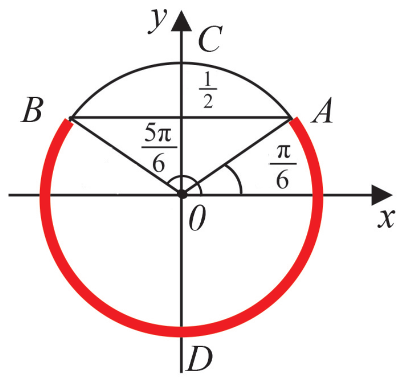
161. 1) $\sin 4x=\sin x$; | 2) $\sin 2x=\cos 3x$; | 3) $\operatorname{tg}10x=\operatorname{tg}8x$; | 4) $\sin 5x=\sin 7x$.

62-64 ENG SODDA TRIGONOMETRIK TENGSIZLIKLAR

$a_1 < \sin x < b_1$, $a_2 < \cos x < b_2$, $a_3 < \mathrm{tg}x < b_3$ ko'rinishdagi tengsizliklar eng sodda trigonometrik tengsizliklar deyiladi. Bu yerda $a_1, b_1, a_2, b_2, a_3, b_3$ – berilgan haqiqiy sonlar. Bunday tengsizliklarni yechishda birlik doiradan, funksiya grafigidan foydalanish qulay.

1- misol. $\sin x \le 0{,}5$ tengsizlikni $[0, 2\pi]$ kesmada yeching.

△Birlik doirani qaraymiz. Bu doirada ordinatalari 0,5 ga teng va undan kichik nuqtalarni topamiz. 20- rasmdan ravshanki, *BDA* yoyning barcha nuqtalari yuqoridagi shartni qanoatlantiradi. Shuning uchun x sonlarning $\left[0;\frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6};2\pi\right]$ to'plami tengsizlikning yechimi bo'ladi. *Javob*: $x\in\left[0;\frac{\pi}{6}\right]\cup\left[\frac{5\pi}{6};2\pi\right]$ ▲

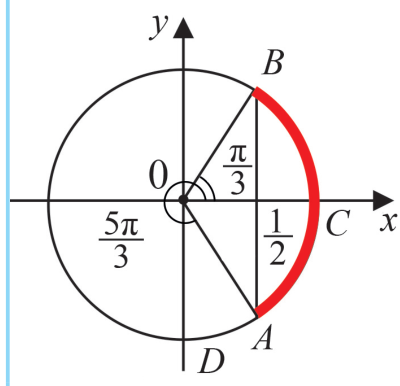


20- rasm.

2- misol. $\cos x > \frac{1}{2}$ tengsizlikni $[0, 2\pi]$ kesmada yeching.

△Birlik doirada abssissalari $\frac{1}{2}$ ga teng va undan katta nuqtalarni topamiz. 21- rasmdan ko'rinb turibdiki, *ACB* yoyning barcha nuqtalari yuqoridagi shartni qanoatlantiradi. Shuning uchun x larning $\left[0;\frac{\pi}{3}\right)\cup\left(\frac{5\pi}{3};2\pi\right]$ to'plami tengsizlikning yechimi bo'ladi.

Javob: $x\in\left[0;\frac{\pi}{3}\right)\cup\left(\frac{5\pi}{3};2\pi\right]$ ▲



21- rasm.

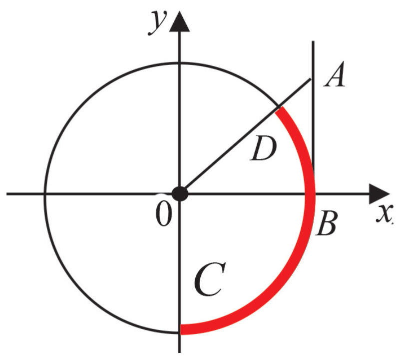
3- misol. $\mathrm{tg}x \le 1$ tengsizlikni $\left(-\frac{\pi}{2};\frac{\pi}{2}\right)$ oraliqda yeching.

△Birlik doiraning *B* nuqtasidan *Oy* o'qiga parallel *AB* to'g'ri chiziq o'tkamiz (22- rasm).

Unda A nuqtani shunday tanlaymizki, bunda $OB=AB$ bo‘lsin. $\triangle\, AOB$ teng yonli va to‘g‘ri burchaklidir. OA gipotenuzaning aylana bilan kesishuv nuqtasi D bo‘lsin.

Rasmdan ravshanki, DBC yoyning barcha nuqtalari berilgan tengsizlikni qanoatlantiradi.

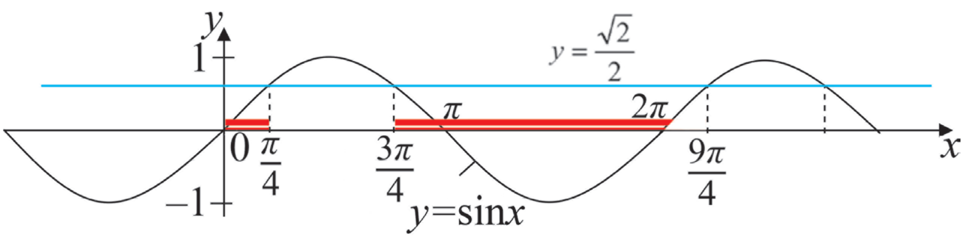
Javob: $x\in\left(-\dfrac{\pi}{2};\dfrac{\pi}{4}\right]$. ▲



22- rasm.

4- misol. Tengsizlikni yeching: $\sin x<\dfrac{\sqrt{2}}{2}$.

△ Bitta koordinatalar sistemasiga $y=\sin x$ va $y=\dfrac{\sqrt{2}}{2}$ (23- rasm) funksiyalar



23- rasm.

grafiklarini chizib, $\sin x=\dfrac{\sqrt{2}}{2}$ tenglamaning [0; 2π] kesmadagi yechimini topamiz. Rasmdan ko‘rinadiki, $\sin x<\dfrac{\sqrt{2}}{2}$ tengsizlikning [0; 2π] kesmadagi yechimi $\left(0;\dfrac{\pi}{4}\right]$ va $\left(\dfrac{3\pi}{4};2\pi\right]$ oraliqlar bo‘ladi. Funksiyaning davriyligidan x ning $\left[2\pi n;\dfrac{\pi}{4}+2\pi n\right)\cup\left(\dfrac{3\pi}{4}+2\pi n;2\pi(n+1)\right]$, $n\in Z$ to‘plami tengsizlikning yechimi bo‘ladi. ▲

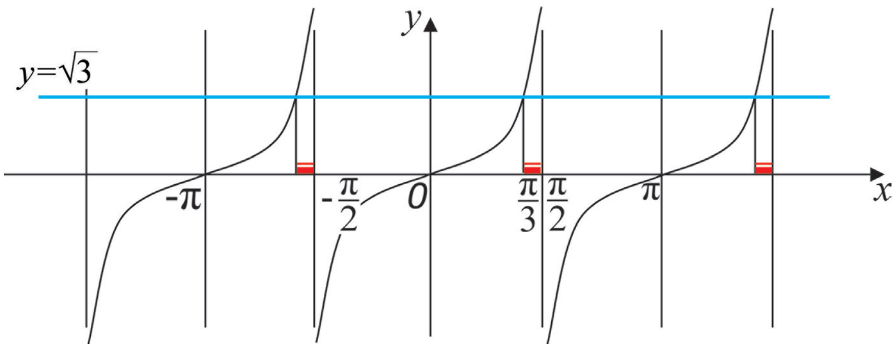
5-misol. Tengsizlikni yeching: $-2\cos x\geq 1$.

△ Avval $y=\cos x$ va $y=-\frac{1}{2}$ funksiyalar grafigini bitta koordinatalar sistemasiga chizamiz. Kegin $\cos x=-\frac{1}{2}$ tenglamaning [0; 2π] kesmadagi yechimlari $\dfrac{2\pi}{3}$ va $\dfrac{4\pi}{3}$ ekanini aniqlaymiz. Demak, tengsizlikning yechimlari $\left[\dfrac{2\pi}{3}+2\pi n;\dfrac{4\pi}{3}+2\pi n\right]$, $n\in Z$ kesmalardan iborat ekan.▲

6- misol. Tengsizlikni yeching: $\operatorname{tg} x \geq \sqrt{3}$.

△ $y=\operatorname{tg} x$ va $y=\sqrt{3}$ funksiyalar grafigini bitta koordinatalar sistemasiga chizamiz (24- rasm). $\operatorname{tg} x=\sqrt{3}$ tenglamani [0, π] kesmadagi yechimini topamiz. Bu tenglamaning yechimi $x=\dfrac{\pi}{3}$. Shuning uchun tengsizlikning [0, π] kesmadagi yechimlari to‘plami $\left[\dfrac{\pi}{3};\dfrac{\pi}{2}\right)$ oraliqdir. y=tg$x$ funksiyaning davri π ekanidan foydalanib, tengsizlikning barcha yechimlarini topamiz:

$$\left[\frac{\pi}{3}+\pi n;\frac{\pi}{2}+\pi n\right),\ n\in Z.$$ ▲



24- rasm.

Savol va topshiriqlar

$\sin x>\dfrac{\sqrt{3}}{2}$, $\cos x>-\dfrac{\sqrt{3}}{2}$, tg$x>-1$ tengsizliklar qanday yechiladi?

Mashqlar

162. Tengsizlikni berilgan oraliqda yeching:

1) $\sin x>\dfrac{1}{2},\ x\in[0;\pi]$;

2) $\cos x>\dfrac{\sqrt{2}}{2},\ x\in\left[-\dfrac{\pi}{2};\dfrac{\pi}{2}\right]$;

3) $\operatorname{tg} x>-\sqrt{3}, x\in\left(-\dfrac{\pi}{2};\dfrac{\pi}{2}\right)$;

4) $\cos x>\dfrac{1}{2},\ x\in\left[-\dfrac{\pi}{2};\dfrac{\pi}{2}\right]$;

5) $\sin x \le \frac{\sqrt{3}}{2},\ x \in [-\pi;0]$; 6 | 6) $\operatorname{tg} x < \frac{1}{\sqrt{3}},\ x \in \left(-\frac{\pi}{2};\frac{\pi}{2}\right)$;

7) $\cos x < -\frac{\sqrt{3}}{2},\ x \in \left[\frac{\pi}{2};\frac{3\pi}{2}\right]$; | 8) $\cos 2x \le \frac{\sqrt{2}}{2},\ x \in \left[\frac{\pi}{2};\frac{3\pi}{2}\right]$.

Tengsizlikni yeching (**163–169**):

163. 1) $\sin x \ge \frac{\sqrt{2}}{2}$; | 2) $\cos\ < -\frac{\sqrt{2}}{2}$; | 3) $\operatorname{tg} x < -\frac{1}{\sqrt{3}}$; | 4) $\sin x < -\frac{\sqrt{3}}{2}$.

164. 1) $\sin x > \frac{1}{2}$; | 2) $\operatorname{tg} x > -1$; | 3) $\cos x \le -\frac{\sqrt{2}}{2}$; | 4) $\cos x \le \frac{1}{2}$.

165. 1) $\sin 3x < \frac{1}{2}$; | 2) $\sin\frac{x}{4} < -\frac{\sqrt{3}}{2}$; | 3) $\cos\frac{x}{2} > \frac{\sqrt{3}}{2}$; | 4) $\operatorname{tg} 3x > 1$.

166. 1) $2\cos(2x + \frac{\pi}{3}) \le \sqrt{2}$; | 2) $\sqrt{2}\sin\left(\frac{x}{2} + \frac{\pi}{6}\right) \ge 1$; | 3) $2\cos(2x - \frac{\pi}{3}) > \sqrt{3}$.

167. 1) $\sin 2x\cos\frac{\pi}{3} - \cos 2x\sin\frac{\pi}{3} \le \frac{\sqrt{3}}{2}$; | 2) $2\sin 2x\cos 2x \ge \frac{1}{2}$.

168. 1) $\sin\frac{\pi}{4}\cos 3x + \cos\frac{\pi}{4}\sin 3x < \frac{\sqrt{2}}{2}$; | 2) $\cos\frac{\pi}{4}\cos 2x - \sin 2x\sin\frac{\pi}{4} < -\frac{\sqrt{3}}{2}$.

169. 1) $\cos\left(\frac{x}{2} + 1\right) \ge \frac{1}{2}$; | 2) $\sin\left(\frac{x}{4} - 2\right) < \frac{\sqrt{2}}{2}$; | 3) $\cos\left(1 - \frac{x}{3}\right) \ge \frac{\sqrt{2}}{2}$.

Nazorat ishi namunasi

Tenglamalarni yeching (**1–4**):

1. $\sin 3x = 0$.
2. $4\cos 6x = -2\sqrt{3}$.
3. $5 \cdot \operatorname{tg} 4x = 3$.
4. $5\operatorname{tg}^2 x - 4\operatorname{tg} x - 1 = 0$.

!

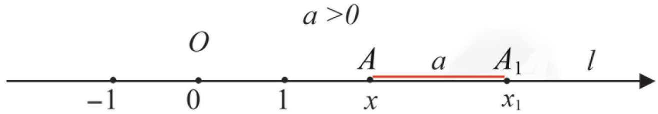
Tengsizliklarni $x \in [0;\ \pi]$ oraliqda yeching (**5-6**):

5. $\sin x > \frac{1}{2}$.
6. $\operatorname{tg} x \le -1$.

68 GRAFIKLARNI ALMASHTIRISH

Siljitish

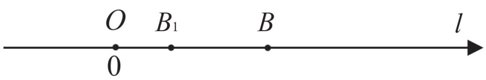
l son o'qi va O nuqta undagi hisob boshi bo'lsin (25- rasm). l ning har qaysi nuqtasi a birlik siljitilsin. Agar $a>0$ bo'lsa, siljitish musbat yo'nalishda (o'q yo'nalishida) bo'ladi. Agar $a<0$ bo'lsa, siljitish qarama-qarshi yo'nalishda bajariladi, $a=0$ da nuqtalar o'z joyidan siljimaydi. Agar x koordinatali $A=A(x)$ nuqta a birlikka siljitilganda $A_1(x_1)$ nuqtaga o'tgan bo'lsa, A_1 nuqtaning koordinatasi $x_1=x+a$ formula bo'yicha aniqlanadi. A nuqta A_1 nuqtaning asli (proobrazi), A_1 esa A ning nusxasi (obrazi) deyiladi.



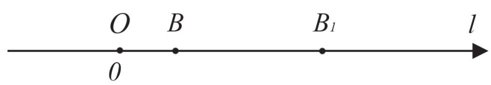
25- rasm.

Cho'zish

l son o'qida $B(x)$ nuqta O koordinata boshidan k marta uzoqlashtirilib (yoki O ga yaqinlashtirilib), $B_1(x)$ nuqtaga o'tkazilgan bo'lsin. B_1 nuqtaning koordinatasi $x_1=kx$ formula bo'yicha hisoblanadi. Agar $k>0$ bo'lsa, B_1 va B nuqtalar O nuqtaning bir tomonida; agar $k<0$ da B_1 va B nuqtalar O ning turli tomonida joylashadi. Agar $|k|<1$ bo'lsa, (26- rasm) $x=OB$ kesma k marta qisqaradi; agar $|k|>1$ bo'lsa, (27- rasm) OB kesma k marta cho'ziladi, $k=1$ da B va B_1 nuqtalar ustma–ust tushadi, $k=-1$ da ular O nuqtaga nisbatan simmetrik joylashadi.



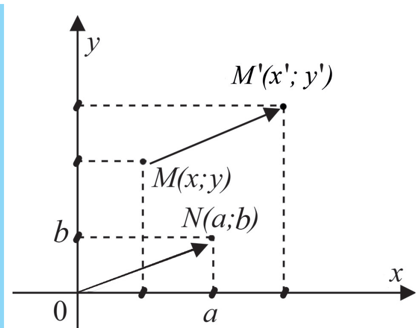
$0<k<1$ 26- rasm.



$k>1$ 27-rasm.

Parallel ko'chirish

Parallel ko'chirishda xOy koordinata tekisligidagi barcha nuqtalar bir xil yo'nalishda bir xil masofaga ko'chadi (28- rasm). Chunonchi, $O(0;\ 0)$ koordinata boshi $N(a;\ b)$ nuqtaga ko'chirilgan bo'lsa, $M\ (x;\ y)$ nuqta $M'\ (x';\ y')$ ga ko'chadi. $M'\ (x';\ y')$ nuqtaning koordinatalari uchun quyidagi formula o'rinli: $x'=x+a$, $y'=y+b$.



28- rasm.

Funksiya grafigini almashtirish

Yuqoridagi almashtirishlar (siljitish, cho'zish, parallel ko'chirish) $y=f(x)$ funksiya grafigi yordamida $y=f(x-a)+b$, $y=m\cdot f\left(\frac{x}{k}\right)$(bunda a, b, m, k – o'zgarmas sonlar va $m\neq 0$, $k\neq 0$) funksiyalar grafigini chizish imkonini beradi.

Masalan, $y=f(x-a)+b$ funksiya grafigini $y=f(x)$ funksiya grafigi yordamida chizish uchun $y=f(x)$ funksiya grafigining har bir nuqtasi a birlik o'ngga siljitiladi va b birlik yuqoriga ko'tariladi, ya'ni (a; b) vektor bo'yicha parallel ko'chiriladi.

$y=f(x)$ funksiya grafigi yordamida $y=m\cdot f\left(\frac{x}{k}\right)$ funksiya grafigini chizish uchun $y=f(x)$ funksiya grafigining har bir nuqtasi abssissasi Ox bo'ylab k marta siqiladi ($k>0$ bo'lsa – o'ngga, $k<0$ bo'lsa – chapga) va ordinatasi Oy o'q bo'ylab m birlik cho'ziladi ($m>0$ bo'lsa – yuqoriga, $m<0$ bo'lsa – pastga).

1- misol. $y=3x$ funksiya grafigi yordamida $y=3(x-1)+4$ funksiya grafigini chizing.

△ $y=3(x-1)+4$ funksiya grafigini chizish uchun $y=3x$ funksiya grafigi (1; 4) vektor bo'yicha parallel ko'chiriladi. ▲

2- misol. $y=-2x+4$ funksiya grafigi yordamida $y=-2(x+3)+5$ funksiya grafigini chizing.

△ $y=-2(x+3)+5$ funksiya grafigini chizish uchun $y=-2x+4$ funksiya grafigi (3; 1) vektor bo'yicha parallel ko'chiriladi. ▲

3- misol. $y=x^2$ parabola grafigidan foydalanib $y=2-(x+3)^2$ funskiya grafigini chizing.

△ $y=2-(x+3)^2$ funksiya grafigini chizish uchun $y=x^2$ funksiya grafigi avval 3 birlik chapga siljitiladi va Ox o'qiga nisbatan simmetrik ko'chiriladi. So'ngra hosil bo'lgan grafik Oy o'qi bo'yicha 2 birlik yuqoriga ko'tariladi. ▲

4- misol. $y=\sin x$ funksiya grafigi yordamida $y=\sin 2x$ funksiya grafigini chizing.

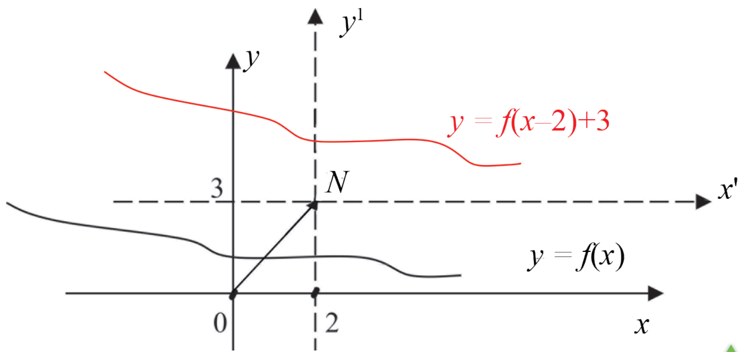
△$y=\sin 2x$ funksiya grafigini chizish uchun $y=\sin x$ funksiya grafigining har bir nuqtasining abssissasi Ox o'qi bo'ylab ikki marta o'ngga siqiladi. ▲

5- misol. $y=\cos x$ funksiya grafigi yordamida $y=-2\cos\left(2x-\frac{\pi}{4}\right)$ funksiya grafigini chizing.

△ $y=-2\cos\left(2x-\frac{\pi}{4}\right)$ yoki $y=-2\cos 2\left(x-\frac{\pi}{8}\right)$ funksiya grafigini chizish uchun avval $y=\cos x$ funksiya grafigi o'ngga $\frac{\pi}{8}$ ga siljitiladi, keyin abssissasi o'ngga ikki martta siqiladi, ordinatasi ikki marta yuqoriga cho'ziladi. So'ngra oxirgi grafik Ox o'qi bo'yicha simmetrik ko'chiriladi. ▲

6- misol. $y=f(x)$ funksiya grafigi yordamida $y=f(x-2)+3$, funksiya grafigini chizing.

△ $y=f(x-2)+3$ funksiya grafigini chizish uchun $y=f(x)$ funksiya grafigining har bir nuqtasi (2; 3) vektor bo'yicha parallel ko'chiriladi (29- rasm). ▲

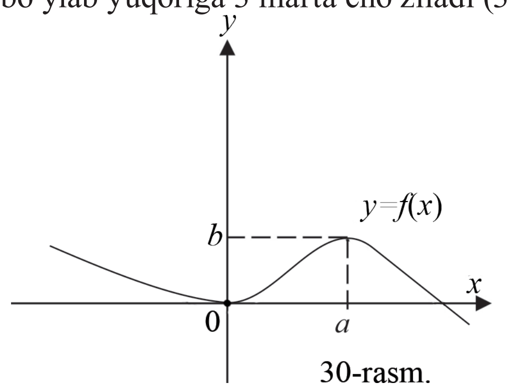


29- rasm.

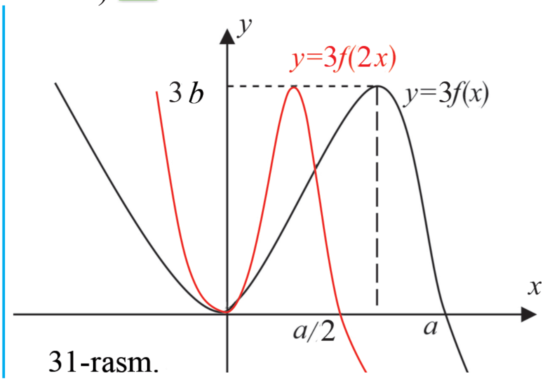
▲

7- misol. $y=f(x)$ funksiya grafigi yordamida (30- rasm) $y=3f(2x)$ funksiya grafigini chizing ($m=3, \quad k=\frac{1}{2}$ bo'lgan hol).

△ $y=f(x)$ funksiya grafigi Ox o'q bo'ylab o'ngga 2 marta siqiladi va Oy o'q bo'ylab yuqoriga 3 marta cho'ziladi (31- rasm). ▲



30-rasm.



31-rasm.

Savol va topshiriqlar

1. Siljitish nima? Cho'zish-chi? Parallel ko'chirish-chi? Misollar keltiring.

4. $y=\sin x$ funksiya grafigi yordamida $y=-\sin\left(x-\frac{\pi}{3}\right)$ funksiya grafigini chizing.

Mashqlar

170. $y=f(x)=x^2-2x+3$ funksiya grafigi yordamida ko‘rsatilgan funksiyalar grafigini chizing:

1) $y=f(x)+1$; 2) $y=3f(x)$; 3) $y=3f(x)-2$;

4) $y=f(x-1)+1$; 5) $y=2f(x-1)+1$; 6) $y=f\left(\frac{x}{2}\right)$;

7) $y=\frac{1}{2}f(2x)$; 8) $y=f(2x)-3$; 9) $y=2f(2x)-5$.

171. $y=f(x)=x^2-5x+6$ funksiya grafigi yordamida ko‘rsatilgan funksiyalar grafigini chizing:

1) $y=f(x-1)$; 2) $y=f\left(\frac{x}{3}\right)$; 3) $y=f(2x)$; 4) $y=3f\left(\frac{x}{3}\right)+1$;

5) $y=-f(x)$; 6) $y=2f(x)-3$; 7) $y=-f(-x)$; 8) $y=2f(x-1)+5$.

172. $y=\cos x$ funksiya grafigi yordamida ko‘rsatilgan funksiyalar grafigini chizing:

1) $y=\cos x-1$; 2) $y=2\cos x+1$;

3) $y=-\cos\left(x+\frac{\pi}{4}\right)$; 4) $y=3\cos\left(2x-\frac{\pi}{3}\right)$.

69-70 PARAMETRIK KO‘RINISHDA BERILGAN SODDA FUNKSIYALARNING GRAFIKLARI

Moddiy nuqtaning (x, y) koordinatlari t parametrga bog‘liq bo‘lsin: $x=\varphi(t)$, $y=\psi(t)$. t biror T oraliqda o‘zgarganda $(\varphi(t), \psi(t))$ nuqtalar to‘plami qanday bo‘ladi? Bu to‘plamni *parametrik ko‘rinishda berilgan funksiyaning grafigi* deb ataymiz.

1- misol. Moddiy nuqtaning koordinatalari parametrik ko‘rinishda $\begin{cases} x=3t+1, \\ y=5t+8 \end{cases}$ berilgan. Bu moddiy nuqta harakati davomida chizgan chiziqni (moddiy nuqta trayektoriyasini) toping.

△ Tenglamalardan t parametrni topamiz: $t=\frac{x-1}{3}$ va $t=\frac{y-8}{5}$.

Hosil bo‘lgan ifodalardan $\frac{x-1}{3}=\frac{y-8}{5}$ tenglamaga kelamiz. Bundan $5x-5=3y-24$, yoki $5x-3y+19=0$. Bu to‘g‘ri chiziqning tenglamasidir.

Demak, izlangan funksiya $3y=5x+19$, yoki $y=\frac{5}{3}x+\frac{19}{3}$ ekan.

Javob: $y=\frac{5}{3}x+\frac{19}{3}$. ▲

2- misol. $\begin{cases} x=3+5\sin t, \\ y=-7+5\cos t \end{cases}$ parametrik ko‘rinishda berilgan funksiya grafigi qanday chiziq bo‘ladi?

△Berilgan tengliklardan $\sin t=\frac{x-3}{5}$, $\cos t=\frac{y+7}{5}$ ekanini topamiz.

$\sin^2 t+\cos^2 t=1$ ayniyatdan foydalanib, $\left(\frac{x-3}{5}\right)^2+\left(\frac{y+7}{5}\right)^2=1$ tenglamaga kelamiz. Bundan $(x-3)^2+(y+7)^2=25$. Bu tenglama markazi $(3;\ -7)$ va radiusi $r=5$ bo‘lgan aylana tenlamasidir.▲

3- misol. Moddiy nuqta koordinatalari $x=7t^2+1$, va $y=3t$ qonuniyat bilan o‘zgarsa, x va y orasidagi bog‘lanishni aniqlang, ya’ni $t\geq 0$.

△Berilgan qonuniyatlardan t ni topamiz: $t=\sqrt{\frac{x-1}{7}}$, $t=\frac{y}{3}$. Bu ifodalardan $\frac{y}{3}=\sqrt{\frac{x-1}{7}}$ tenglamaga kelamiz. Budan oxirida $y=3\sqrt{\frac{x-1}{7}}$ funksiyani topamiz. Demak, izlangan funksiya $y=3\sqrt{\frac{x-1}{7}}$ ekan. ▲

4- misol. $\begin{cases} x=4\sin t, \\ y=3\cos t \end{cases}$ parametrik ko‘rinishda berilgan funksiya grafigi qanday chiziq bo‘ladi, bu yerda $0\leq t\leq 2\pi$?

△Berilgan tengliklardan $\sin t=\frac{x}{4}$ va $\cos t=\frac{y}{3}$ ekanini topamiz. $\sin^2 t+\cos^2 t=1$ ayniyatdan foydalanib, $(\frac{x}{4})^2+(\frac{y}{3})^2=1$, yoki $\frac{x^2}{16}+\frac{y^2}{9}=1$ tenglamani hosil qilamiz. Bu tenglama bilan berilgan nuqtalar to‘plami markazi koordinata boshida va yarim o‘qlari $a=4$, $b=3$ bo‘lgan *ellips* deb nomlanadi. ▲

Savol va topshiriqlar

Parametrik ko‘rinishda berilgan funksiyalarga misollar keltiring.

Mashqlar

173. Moddiy nuqtaning koordinatalari parametrik ko'rinishda berilgan. Bu moddiy nuqta harakati davomida chizgan chiziqning (moddiy nuqta trayektoriyasining) formulasini toping. Mos rasm chizing:

1) $\begin{cases} x = 2t+1, \\ y = 4t+8; \end{cases}$ 2) $\begin{cases} x = 6t+4, \\ y = 9t+3; \end{cases}$ 3) $\begin{cases} x = 4t+9, \\ y = 7t+18; \end{cases}$ 4) $\begin{cases} x = 12t+11, \\ y = 15t+18. \end{cases}$

174. Moddiy nuqta koordinatalari parametrik ko'rinishda berilgan. x va y koordinatalar orasidagi bog'lanishni aniqlang:

1) $\begin{cases} x = 17t^2+1, \\ y = 13t; \end{cases}$ 2) $\begin{cases} x = 27t^2+21, \\ y = 23t; \end{cases}$ 3) $\begin{cases} x = 37t^2+31, \\ y = 33t; \end{cases}$ 4) $\begin{cases} x = 47t^2+41, \\ y = 43t. \end{cases}$

175. Parametrik ko'rinishda berilgan funksiya grafigi qanday chiziqdan iborat? Mos rasmni chizing:

1) $\begin{cases} x = 7\sin t, \\ y = 7\cos t, \\ 0 \le t \le 2\pi; \end{cases}$ 2) $\begin{cases} x = \sin t, \\ y = \cos t, \\ 0 \le t \le 2\pi; \end{cases}$ 3) $\begin{cases} x = 5\sin t, \\ y = 5\cos t, \\ 0 \le t \le 2\pi; \end{cases}$ 4) $\begin{cases} x = 9\sin t, \\ y = 9\cos t, \\ 0 \le t \le 2\pi. \end{cases}$

176. Parametrik ko'rinishda berilgan funksiya grafigi qanday chiziqdan iborat? Mos rasmni chizing:

1) $\begin{cases} x = 6\sin t+3, \\ y = 6\cos t+7, \\ 0 \le t \le 2\pi; \end{cases}$ 2) $\begin{cases} x = 3\sin t, \\ y = 3\cos t-1, \\ 0 \le t \le 2\pi; \end{cases}$ 3) $\begin{cases} x = 2\sin t-3, \\ y = 2\cos t+7, \\ 0 \le t \le 2\pi. \end{cases}$

71 KO'RSATKICHLI FUNKSIYA VA UNING GRAFIGI

Daraja va uning xossalari

Haqiqiy son ko'rsatkichli daraja quyidagi xossalarga ega ($a>0,\ a\neq1$):

1) $a^x \cdot a^y = a^{x+y}$; 2) $a^x : a^y = a^{x-y}$; 3) $(a^x)^y = a^{x\cdot y}$;

4) $(a\cdot b)^x = a^x \cdot b^x$; 5) $\left(\dfrac{a}{b}\right)^x = \dfrac{a^x}{b^x}$;

6) agar $0<a<b$ va $x>0$ bo'lsa, $a^x<b^x$; 7) agar $0<a<b$ va $x<0$ bo'lsa, $a^x>b^x$;
8) agar $x<y$ va $a>1$ bo'lsa, $a^x<a^y$; 9) agar $x<y$ va $0<a<1$ bo'lsa, $a^x>a^y$ bo'ladi.

1- misol. Taqqoslang: $2^{-\sqrt{3}}$ va $3^{-\sqrt{3}}$.

△ 7- xossaga ko'ra $0<2<3$ va $-\sqrt{3}<0$ bo'lgani uchun $2^{-\sqrt{3}} > 3^{-\sqrt{3}}$. ▲

2- misol. Taqqoslang: $\left(\frac{1}{2}\right)^{0,2}$ va $\left(\frac{1}{2}\right)^{0,3}$.

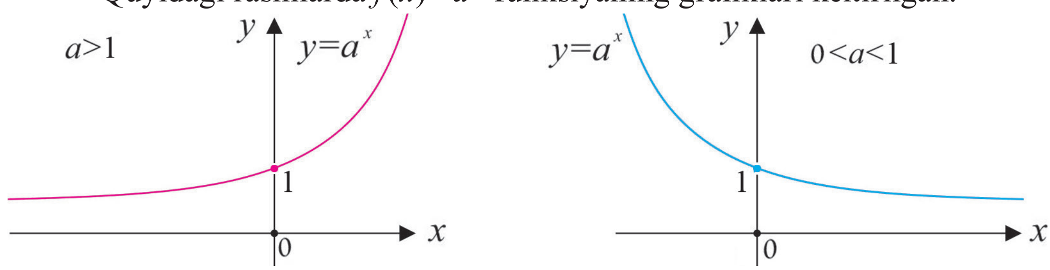
△ 9- xossaga ko‘ra 0,2<0,3 va $0<\frac{1}{2}<1$ bo‘lgani uchun $\left(\frac{1}{2}\right)^{0,2}>\left(\frac{1}{2}\right)^{0,3}$. ▲

Ko‘rsatkichli funksiya va uning xossalari

$f(x)=a^x$, $a>0$, $a\neq1$ ko‘rinishdagi funksiya ko‘rsatkichli funksiya deyiladi. Bunday funksiya quyidagi xossalarga ega:

1) aniqlanish sohasi $(-\infty;\ +\infty)$ oraliqdan iborat;
2) qiymatlar sohasi $(0;\ +\infty)$ oraliqdan iborat;
3) barcha $a(a>0,\ a\neq1)$ uchun $a^0=1$;
4) $a>1$ bo‘lsa, funksiya o‘suvchi;
5) $0<a<1$ bo‘lsa, funksiya kamayuvchidir.

Quyidagi rasmlarda $f(x)=a^x$ funksiyaning grafiklari keltirilgan.



Savol va topshiriqlar

1. Haqiqiy son ko‘rsatkichli darajaning xossalarini ayting. Misollar keltiring.
2. Ko‘rsatkichli funksiyaning xossalarini ayting.

Mashqlar

177. Hisoblang:

1) $\left((\sqrt{3})^{\sqrt{2}}\right)^{\sqrt{2}}$; 2) $9^{\sqrt{3}}:3^{2\sqrt{3}}$; 3) $\left(2^{\sqrt[3]{4}}\right)^{\sqrt[3]{2}}$; 4) $4^{6\sqrt{2}-1}\cdot16^{1-3\sqrt{2}}$.

Taqqoslang (**178–179**):

178. 1) $2^{-\sqrt{3}}$ va 1; 2) $4^{-\sqrt{6}}$ va $(\frac{1}{2})^4$; 3) $(\frac{1}{3})^{\sqrt{5}}$ va 1.

179. 1) $-3^{\sqrt{2}}$ va 1; 2) $(\frac{1}{2})^{-\sqrt{2}}$ va $(\frac{1}{3})^{-\sqrt{2}}$; 3) $(\frac{1}{2})^{\sqrt{2}}$ va $(\frac{1}{3})^{\sqrt{2}}$.

180. Funksiyalarning o‘suvchi yoki kamayuvchi ekanini aniqlang (**180–182**):

1) $y=4^x$; 2) $y=-3^x$; 3) $y=5^x-2$; 4) $y=-(\frac{1}{2})^x+1$.

181. 1) $y=\sqrt{3}^x$; 2) $y=(\frac{1}{\sqrt{3}})^x$; 3) $y=(\frac{\pi}{3})^x$; 4) $y=(\sqrt{3}-1)^x$.

182. 1) $y=(\sqrt{3}-1)^{-x}$; 2) $y=(\sqrt{10}-2)^x$; 3) $y=\left(\pi-\sqrt{2}\right)^x-3$.

72-74 BEVOSITA YECHILADIGAN KO'RSATKICHLI TENGSIZLIKLAR

$a^{f(x)}>a^{g(x)}$, $a>0$, $a\neq 1$ ko'rinishdagi tengsizlik

$a^{f(x)}>a^{g(x)}$, $a>0$, $a\neq 1$ tengsizlik ko'rsatkichli tengsizlikka misol bo'la oladi. Bu tengsizlik $a>1$ bo'lganda $f(x)>g(x)$ tengsizlikka, $0<a<1$ bo'lganda esa $f(x)<g(x)$ tengsizlikga tengkuchlidir.

1- misol. Tengsizlikni yeching: $3^{x+5}>3^{2-5x}$.

△ $a=3>1$ bo'lgani uchun berilgan tengsizlik $x+5>2-5x$ tengsizlikga tengkuchli. Bundan $6x>-3$, yoki $x>-0{,}5$ ekanini topamiz. Demak, tengsizlikning yechimi $(-0{,}5;\ \infty)$ oraliqdan iborat. *Javob*: $x\in(-0{,}5;\ \infty)$. ▲

2- misol. Tengsizlikni yeching: $2\cdot3^{x+2}-2\cdot3^{x+1}-5\cdot3^x<63$.

△ 3^x ni qavsdan tashqariga chiqaramiz: $3^x(2\cdot3^2-2\cdot3-5\cdot1)<63$. Soddalashtirib, $3^x<9$ tengsizlikni hosil qilamiz. Bundan $x<2$. *Javob*: $x\in(-\infty;\ 2)$. ▲

3- misol. Tengsizlikni yeching: $8^{5x^2-46}\geq 8^{2(x^2+1)}$.

△ $a=8>1$ bo'lgani uchun tengsizlik $5x^2-46\geq2(x^2+1)$ tengsizlikka tengkuchli. Shu tengsizlikni yechamiz: $3x^2\geq48$, bundan $x^2\geq16$. Demak, berilgan tengsizlikning yechimi $x\in(-\infty;\ -4]\cup[4;\ +\infty)$ bo'ladi. ▲

$a^x<b$ tengsizlikning ($a>0$, $a\neq1$) $b<0$ bo'lganda yechimi yo'q va $a^x>b$ tengsizlikning $b<0$ bo'lganda yechimi $(-\infty;+\infty)$ oraliqdan iborat ekanligi ravshan.

4- misol. Tengsizlikni yeching: $4^x+2^x-6\geq0$.

△ $2^x=t$ almashtirish kiritamiz, natijada $t^2+t-6\geq0$ kvadrat tengsizlik hosil bo'ladi. Bundan $t\leq-3$, $t\geq2$ ekanini topamiz. Undan $2^x\geq2$ va $2^x\leq-3$ tengsizliklarga kelamiz. 1- tengsizlikdan $x\geq1$ yechim topiladi, 2- tengsizlikning esa yechimi yo'q. Demak berilgan tengsizlikning yechimi $[1;\ +\infty;)$ oraliqdan iborat. *Javob*: $x\in[1;\ +\infty)$. ▲

Savol va topshiriqlar

? $a^{f(x)}>a^{g(x)}$, $a>0$, $a\neq1$ tengsizlik haqida ma'lumot bering. Misol keltiring.

Mashqlar

183. Tengsizlikni yeching (**183–184**):

1) $4^{3x+5} \le 4^{3-5x}$; 2) $7^{4x+5} < 7^{9-5x}$; 3) $6^{x+5} > 6^{3x}$, 4) $8^{x+5} \le 8^{2-5x}$;
5) $11^{x} < 11^{2+5x}$; 6) $2^{x-5} > 2^{25x}$; 7) $2\cdot 2^{x+2} - 3\cdot 2^{x+1} - 5\cdot 2^{x} \le -6$;
8) $3\cdot 5^{x+3} - 5^{x+2} - 2\cdot 5^{x+1} < 68$; 9) $2\cdot 4^{x+2} + 4^{x+1} - 5\cdot 4^{x} \le 31$;
10) $2\cdot 7^{x+2} - 2\cdot 7^{x+1} - 14\cdot 7^{x} < 10$. 11) $13^{x^2+46} \le 13^{x^2+25x}$;
12) $3^{x^2-4x} < 3^{2(x^2-15)}$; 13) $7^{2x^2-4} \le 7^{3(x^2-x)}$;

184. 1) $9^{x} + 3^{x} - 6 \le 84$; 2) $25^{x} + 5^{x} - 30 > 0$;
3) $5\cdot 4^{x} + 2^{x} - 6 \le 0$; 4) $9^{x} + 3^{x} - 12 > 0$;

Nazorat ishi namunasi

1. $\begin{cases} x = 7\sin 5t \\ y = 7\cos 5t \end{cases}$ koʻrinishidagi funksiya grafigini yasang.

2. $y = 11^{x} + 7$ funksiyaning xossalarini yozing.

Tengsizliklarni yeching (**3–5**):

3. $6^{x^2-7x-1} < 6^{7}$.
4. $\left(\frac{1}{2}\right)^{17x} \ge \left(\frac{1}{2}\right)^{54-x}$.
5. $0,7^{-3x} \le 1$.

75-78 LOGARIFM HAQIDA TUSHUNCHA. LOGARIFMIK FUNKSIYA. ENG SODDA LOGARIFMIK TENGLAMA VA TENGSIZLIKLAR

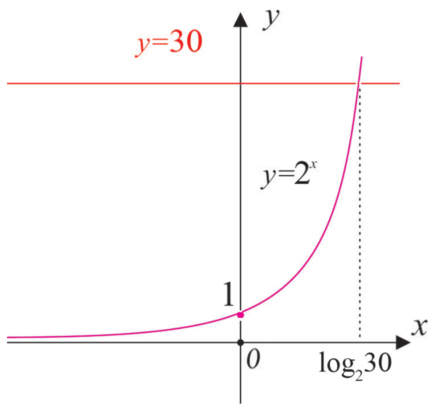
Logarifm haqida tushuncha

$2^{x}=32$ tenglamaning ildizi $x=5$, ammo $2^{x}=30$ tenglamaning ildizi qanday topiladi? Bu kabi tenglamalarni yechish uchun sonning logarifmi tushunchasi kiritiladi. $2^{x}=30$ *tenglama yagona ildizga ega*. Uni 32- rasmdan koʻrish mumkin.

Bu ildiz 30 sonining 2 asosga koʻra logarifmi deyiladi va $\log_2 30$ kabi belgilanadi. Demak, $2^{x}=30$ tenglamaning ildizi $x=\log_2 30$ sondir.

Ushbu ta’rifni kiritamiz:

b musbat sonning *a* asosga koʻra logarifmi deb, *b* sonni hosil qilish uchun asos *a* ni koʻtarish kerak boʻlgan daraja koʻrsatgichiga aytiladi va $\log_a b$ kabi belgilanadi. Asos $a>0$ va $a\ne 1$ shartni qanoatlantirishi kerak.



32- rasm.

Masalan, $\log_3 9=2$, chunki $9=3^2$. Shuningdek, $\log_2 \frac{1}{8}=-3$; $\log_5 5=1$; $\log_7 1=0$.

1- misol. Hisoblang: $\log_3 81$.

△ Logarifmning ta'rifiga ko'ra, $3^4=81$ bo'lgani uchun $\log_3 81=4$. ▲

Logarifmning xossalari

- asosiy logarifmik ayniyat: agar $a>0$, $a\neq 1$, $b>0$ bo'lsa, $a^{\log_a b}=b$ tenglik o'rinlidir;
- agar $a>0$, $a\neq 1$ bo'lsa, $\log_a 1=0$; $\log_a a=1$;
- agar $a>0$, $a\neq 1$ va $x>0$, $y>0$ bo'lsa, $\log_a(xy)=\log_a x+\log_a y$;
- agar $a>0$, $a\neq 1$ va $x>0$, $y>0$ bo'lsa, $\log_a \frac{x}{y} = \log_a x-\log_a y$;
- agar $a>0$, $a\neq 1$, $x>0$ bo'lsa $\log_a x^n = n\cdot\log_a x$;
- yangi asosga (bir asosdan boshqa asosga) o'tish formulasi: agar $a>0$, $a\neq 1$, $x>0$, $b>0$, $b\neq 1$ bo'lsa, $\log_a x=\dfrac{\log_b x}{\log_b a}$;
- agar $a>0$, $a\neq 1$, $b>0$, $b\neq 1$ bo'lsa, $\log_a b\cdot\log_b a=1$.

$\log_{10}x=\lg x$ va $\log_e x=\ln x$ kabi belgilash qabul qilingan. (e=2,718281...).
Bunda $\lg x$ – x ning o'nli logarifmi, $\ln x$ esa x ning natural logarifm deyiladi.
$f(x)=\log_a x$ funksiya (bu yerda x – argument, $a>0$, $a\neq 1$) a – asosli *logarifmik funksiya* deyiladi.

Logarifmik funksiyaning xossalari:

- aniqlanish sohasi $(0; +\infty)$ oraliq;
- qiymatlar sohasi $\mathbb{R}=(-\infty; +\infty)$;
- noli: $x=1$, ya'ni $\log_a 1=0$.
- $a>1$ bo'lsa, logarifmik funksiya $(0; +\infty)$ oraliqda o'suvchi;
- $0<a<1$ bo'lsa, logarifmik funksiya $(0; +\infty)$ oraliqda kamayuvchi.

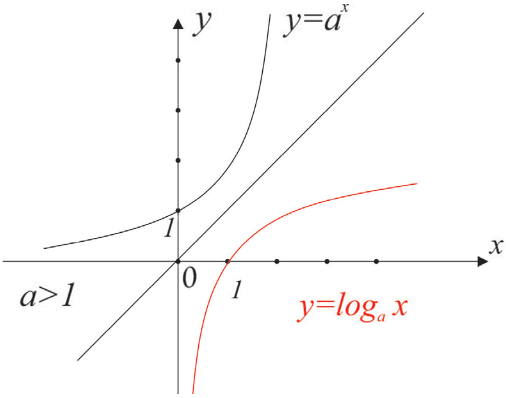
2- misol. Taqqoslang: $\log_{\frac{1}{2}}\frac{1}{3}$ va 0.

△ $\log_{\frac{1}{2}} 1=0$, asos $a=\frac{1}{2}$, ya'ni funksiya kamayuvchi $0<\frac{1}{2}<1$ va $0<\frac{1}{3}<1$ bo'lganidan $\log_{\frac{1}{2}}\frac{1}{3}>\log_{\frac{1}{2}}1$ bo'ladi. Demak, $\log_{\frac{1}{2}}\frac{1}{3}>0$ ekan. ▲

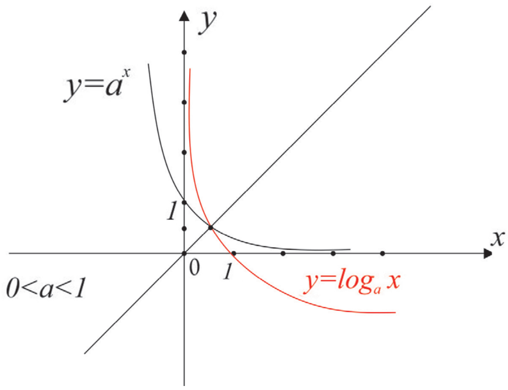
3- misol. Funksiyaning aniqlanish sohasini toping: $f(x)=\log_2\dfrac{x^2-5x+6}{x-1}$.

△ Bu logarifmik funksiyaning aniqlanish sohasi x ning $\dfrac{x^2-5x+6}{x-1}>0$ tengsizlikni qanoatlantiruvchi barcha qiymatlari to‘plamidan iborat. Bu tengsizlikni yechib, funksiyaning aniqlanish sohasi $x \in (1;2) \cup (3;+\infty)$ ekanini topamiz. ▲

33 va 34- rasmlarda $y=a^x$ va $y=\log_a x$ funksiyalarning ($a>1$ va $0<a<1$ hollar uchun) grafiklari birgalikda tasvirlangan.



33- rasm.



34- rasm.

4- misol. Taqqoslang: $\log_3 2 + \log_3 8$ va $\log_3(2+8)$.

△ Logarifmning xossalaridan foydalanamiz: $\log_3 2 + \log_3 8 = \log_3(2 \cdot 8) = \log_3 16$ $\log_3(2+8) = \log_3 10$. Logarifmning asosi $3>1$ bo‘lgani uchun $\log_3 16 > \log_3 10$. Bundan: $\log_3 2 + \log_3 8 > \log_3(2+8)$. ▲

5- misol. Hisoblang: $A = 4^{\log_8 125} + 27^{\frac{1}{3} - \frac{1}{2}\log_3 4}$.

△ Logarifmning xossalaridan foydalanamiz: $\frac{1}{2}\log_3 4 = \log_3 2;$

$$\log_8 125 = \frac{\log_2 125}{\log_2 8} = \frac{3\log_2 5}{3} = \log_2 5. \quad 4^{\log_8 125} = 4^{\log_2 5} = 2^{2\log_2 5} = 2^{\log_2 25} = 25.$$

Shuningdek, $27^{\frac{1}{3} - \frac{1}{2}\log_3 4} = 27^{\frac{1}{3} - \log_3 2} = 27^{\frac{1}{3}} \cdot 27^{-\log_3 2} =$

$= 3 \cdot 3^{-3\log_3 2} = 3 \cdot 3^{\log_3 \frac{1}{8}} = 3 \cdot \frac{1}{8} = \frac{3}{8}$. Demak, $A = 25 + \frac{3}{8} = 25\frac{3}{8}$. ▲

6- misol. Hisoblang: $\dfrac{\lg 54 + \lg \frac{1}{2}}{\lg 72 - \lg 8}$.

△ Logarifmning xossalaridan foydalanamiz:

$$\lg 54 + \lg \frac{1}{2} = \lg(54 \cdot \frac{1}{2}) = \lg 27 = \lg 3^3 = 3\lg 3,$$

$$\lg 72 - \lg 8 = \lg \frac{72}{8} = \lg 9 = \lg 3^2 = 2\lg 3.$$

U holda: $\dfrac{\lg 54+\lg\frac{1}{2}}{\lg 72-\lg 8}=\dfrac{3\lg 3}{2\lg 3}=\dfrac{3}{2}$. *Javob*: $\dfrac{3}{2}$. ▲

Eng sodda logarifmik tenglama

$\log_a x=b$ ko‘rinishdagi tenglamani ($a>0,\ a\neq 1,\ b$ – haqiqiy son) eng sodda logarifmik tenglama deyish mumkin. Tenglamaning yagona yechimi: $x=a^b$.

7- misol. Tenglamani yeching: $\log_3 x=\dfrac{1}{2}$.

△Logarifm ta’rifiga ko‘ra, yechimi $x=3^{\frac{1}{2}}=\sqrt{3}$. *Javob*: $x=\sqrt{3}$. ▲

8- misol. Tenglamani yeching: $\log_x 16=2$.

△Logarifmning ta’rifiga ko‘ra, $x^2=16$ va $x>0,\ x\neq 1$ Demak, tenglamaning yechimi $x=4$ ekan. *Javob*: $x=4$. ▲

9- misol. Tenglamani yeching: $\log_2(x^2-5x+10)=4$.

△Logarifmning ta’rifiga ko‘ra, $x^2-5x+10=2^4$ tenglamani hosil qilamiz. Kvadrat tenglamani yechib $x_1=-1,\ x_2=6$ ildizlarni topamiz. Demak, tenglamaning yechimi $\{-1;\ 6\}$ ekan. *Javob*: $x=-1,\ x=6$. ▲

10- misol. Tenglamani yeching: $\lg(2x-3)=\lg(x-1)$.

△Logarifmning ta’rifiga ko‘ra, $2x-3>0,\ x>1$ bo‘lishi kerak. Tenglamaning aniqlanish sohasi $x>\dfrac{3}{2}$ oraliqdan iborat. Logarifmning xossasiga ko‘ra, $2x-3=x-1$ tenglamaga kelamiz, bundan $x=2$. Bu ildiz esa aniqlanish sohasiga tegishli. *Javob*: $x=2$. ▲

11- misol. Tenglamani yeching: $\log_x(x+2)=2$.

△Tenglamaning aniqlanish sohasini topamiz: $x+2>0\ \ x>0,\ x\neq 1$, ya’ni tenglama $(0,1)\cup(1;\infty)$ to‘plamda aniqlangan. Logarifmning ta’rifiga ko‘ra, $x+2=x^2$ tenglamani hosil qilamiz. Bu kvadrat tenglamani yechib $x_1=-1,\ x_2=2$ ildizlarni topamiz. Bu ildizlardan faqat $x=2$ aniqlanish sohasiga tegishli. Shuning uchun ham u berilgan tenglamaning yechimi bo‘ladi. *Javob*: $x=2$. ▲

12-misol. Tenglamani yeching: $\log_3^2 x-5\log_3 x+6=0$.

△$t=\log_3 x$ belgilash kiritib, $t^2-5t+6=0$ kvadrat tenglamani hosil qilamiz. Uni yechib $t=2$ va $t=3$ ildizlarni topamiz. Topilgan ildizlarni $t=\log_3 x$ ga qo‘yib, $\log_3 x=2$ va $\log_3 x=3$ tengliklarni olamiz. Bu tenglamalarning yechimlari, mos ravishda, 9 va 27 bo‘ladi. *Javob*: $x=9,\ x=27$. ▲

Eng sodda logarifmik tengsizlik

$\log_a x > b$ ko‘rinishdagi tengsizlikni ($a>0,\ a\neq 1,\ b$ – haqiqiy son) eng sodda logarifmik tengsizlik deyish mumkin.

13- misol. Tengsizlikni yeching: $\log_{\frac{1}{2}}(3-x)>-3$.

△ $3-x>0$ bo‘lishi kerak, $-3=\log_{\frac{1}{2}}8$ ekanidan $\log_{\frac{1}{2}}(3-x)>\log_{\frac{1}{2}}8$. Asos $a=\frac{1}{2}<1$ bo‘lgani uchun logarifmik funksiya kamayuvchi, demak, $3-x<8$ va $0<3-x<8$. Bundan $-3<-x<5$ yoki $-5<x<3$ tengsizliklarga kelamiz.
Javob: $x\in(-5;\ 3)$. ▲

14- misol. Tengsizlikni yeching: $\lg(x+1)<\lg(2x-3)$.
△ Logarifmik funksiyaning xossalaridan quyidagi tengsizliklar sistemasini olamiz:

$$\begin{cases} x+1<2x-3, \\ x+1>0, \\ 2x-3>0 \end{cases} \quad \text{yoki} \quad \begin{cases} x>4, \\ x>-1, \\ x>\dfrac{3}{2}. \end{cases}$$

Bu sistemaning yechimi $(4;\ +\infty)$ oraliqdan iborat. *Javob*: $x\in(4;\ +\infty)$. ▲

15- misol. Tengsizlikni yeching: $\log^2_{\frac{1}{2}}x-9\le 0$.

△ Logarifmik funksiya ta’rifiga ko‘ra, $x>0$ bo‘lishi kerak. $t=\log_{\frac{1}{2}}x$ belgini kiritamiz. U holda $t^2-9\le 0$ tengsizlikni hosil qilamiz. Buni yechib $-3\le t\le 3$, ya’ni $-3\le\log_{\frac{1}{2}}x\le 3$ tengsizliklarga kelamiz. $-3=\log_{\frac{1}{2}}8;\ 3=\log_{\frac{1}{2}}\frac{1}{8}$ ekanidan $\log_{\frac{1}{2}}8\le\log_{\frac{1}{2}}x\le\log_{\frac{1}{2}}\frac{1}{8}$. Asos $a=\frac{1}{2}<1$ bo‘lgani uchun $y=\log_{\frac{1}{2}}x$ funksiya kamayuvchi, demak, $\frac{1}{8}\le x\le 8$ bo‘lishi kerak. *Javob*: $x\in[\frac{1}{8};8]$. ▲

Savol va topshiriqlar

1. Logarifmga ta’rif bering. Misol keltiring.
2. Logarifmning xossalarini ayting. Misolda tushuntiring.
3. Logarifmik funksiyalarning xossalarini ayting.
4. Eng sodda logarifmik tenglama nima va u qanday yechiladi?

5. Eng sodda logarifmik tengsizlik nima va u qanday yechiladi? Misol keltiring.

Mashqlar

185. Hisoblang:

1) $\log_5 125$; 2) $\log_{\frac{1}{3}} 9$; 3) $\log_5 0,04$; 4) $\log_{0,1} 1000$; 5) $\log_3 \frac{1}{27}$.

186. Taqqoslang:

1) $\log_2 3$ va $\log_2 5$; 2) $\frac{\log_2 3}{\log_2 5}$ va $\log_5 4$; 3) $\log_{\frac{1}{2}} 3$ va $\log_{\frac{1}{2}} 5$;

4) $\log_2 3$ va 1; 5) $\log_3 2 + \log_3 5$ va $\log_3(2+5)$; 6) $\log_7 \frac{1}{2}$ va 0.

187. Hisoblang:

1) $1,5^{\log_{1,5} 2}$; 2) $e^{\ln 5}$; 3) $2^{3\log_2 5}$; 4) $3^{2+\log_3 5}$; 5) $7^{-2\log_7 6}$;

6) $3^{3-\log_3 54}$; 7) $\log_6 2 + \log_6 18$; 8) $\lg 25 + \lg 4$; 9) $\log_3 \frac{5}{9} + \log_3 \frac{1}{5}$;

10) $\frac{\lg 2 + \lg 162}{2\lg 3 + \lg 2}$; 11) $\log_4 7 - \log_4 \frac{7}{16}$; 12) $\frac{\ln 64}{\ln 4}$.

188. Funksiyalarning aniqlanish sohasini toping:

1) $y = \log_3(2x-5)$; 2) $y = \log_7(x^2-2x-3)$; 3) $y = \log_5(4-x^2)$.

4) $y = \log_2(x^2-2x+1)$; 5) $y = \log_{\sqrt{2}}(3-x)$; 6) $y = \log_2 \frac{x-1}{x+2}$.

189. Funksiyaning grafigini chizing:

1) $y = \log_2 x$; 2) $y = \log_{\frac{1}{3}} x$; 3) $y = \log_4(x-1)$; 4) $y = -\log_3 x$.

190. Tenglamani yeching:

1) $\log_2 x = -5$; 2) $\log_{\sqrt{3}} x = 0$; 3) $\log_{\frac{1}{2}} x = -2$; 4) $\log_x 128 = 7$;

5) $\log_9 x = \frac{1}{2}$; 6) $\log_{\sqrt{x}} 27 = 3$; 7) $\log_3 x = 5$;

8) $\log_2(x-5) = \log_2(4x+1)$; 9) $\log_{\frac{1}{2}} x = -2$; 10) $\log_5(3-2x) = \log_{\frac{1}{5}} x$;

11) $\log_{\frac{1}{3}}(3x-6) = -2$; 12) $\log_2(x+1) + \log_2(8-x) = 3$; 13) $\log_x 5 = 2$;

14) $\lg(x^2+x-10)-\lg(x-3)=1$; 15) $\log_7^2 x - \log_7 x = 2$;

16) $5^{4-x}=6$; 17) $\log_x 3+\log_3 x=2$; 18) $5^{x^2}=6$; 19) $5^{x^2}=\frac{1}{2}$;

20) $\lg(x^2-6x+19)=1$; 21) $\log_5(5^x-4)=1-x$.

191. Tengsizlikni yeching:

1) $\log_8 x>2$; 2) $\log_3^2 x-3>2\cdot\log_3 x$; 3) $\log_8 x<2$; 4) $\log_{\frac{1}{2}} x>1$;

5) $\lg(3-2x)>1$; 6) $2^{x+1}<3$; 7) $\log_3(2x-4)<\log_3(x+1)$; 8) $2^{|x+1|}>3$.

79-81 KO'RSATKICHLI VA LOGARIFMIK FUNKSIYALAR YORDAMIDA MODELLASHTIRISH

1-misol. Bakteriya ma'lum vaqtdan (bir necha soat yoki minutlardan) so'ng ikkiga bo'linadi va bakteriyalar soni ikki karra ortadi. Navbatdagi vaqtdan so'ng mazkur ikkita bakteriya ham ikkiga bo'linadi va populatsiya miqdori (bakteriyalar umumiy soni) yana ikki karra ortadi; endi, bakteriyalar soni to'rtta bo'ldi. Bu ko'payish jarayoni qulay sharoitlarda (populatsiya uchun zarur resurslar: joy, oziqa, suv, energiya va hokazolar mavjud bo'lganda) davom etaveradi.

Faraz qilaylik, dastlab 10 millionta bakteriya borligi, bunday bakteriyalar bir soatdan so'ng ikkiga bo'linishi ma'lum bo'lsin. Quyidagi jadval $t=1, 2, 3, 4$ soat o'tganda b populatsiya miqdori qanday o'zgarishini ifodalaydi:

t (soat)	0	1	2	3	4
b_t (million)	10	20	40	80	160

Shu bilan birga, barcha bakteriyalar ham har soatda bir vaqtda sinxron ravishda ikkiga bo'linmasligini ma'lum. Bunday holatda t butun son bo'lmaganda (masalan, $t=1\frac{1}{2}$ soat) bakteriyalar populatsiyasi miqdorini topish masalasi turibdi.

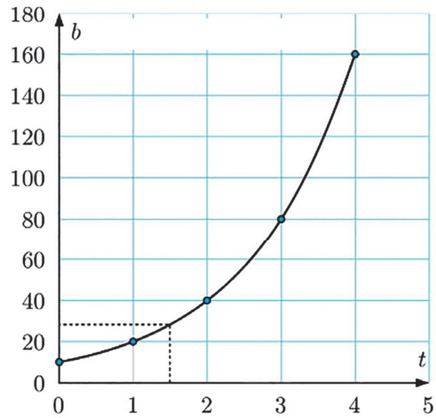
a) $b_1, b_2, \ldots$ ketma-ketlik qanday ketma-ketlik?

b) Tekislikdagi to'g'ri burchakli koordinatalar sistemasida jadval bo'yicha mos nuqtalarni belgilab, so'ng hosil bo'lgan nuqtalarni silliq chiziq bilan tutashtiring.

c) $t=1\frac{1}{2}$ soat o'tgandan keyin bakteriyalar populatsiyasi qanday bo'ladi?

d) Bakteriyalar populatsiyasining ixtiyoriy t vaqtga nisbatan o'zgarishini qanday funksiya yordamida modellashtirsa bo'ladi?

△ Ikkinchi qatordagi $b_1, b_2, \ldots$ sonlar ketma-ketligi mahraji 2 ga teng bo'lgan geometrik progressiya ekanligi ravshan. Uning umumiy ko‘rinishi quyidagicha bo‘ladi: $b_t=20\cdot2^{t-1}$, bu erda $t = 1, 2, 3, 4$.



Tekislikdagi koordinatalar sistemasida jadval bo‘yicha mos nuqtalarni belgilab, so‘ng hosil bo‘lgan nuqtalarni silliq chiziq bilan tutashtiraylik:

$t=1\frac{1}{2}$ soat o‘tganda bakteriyalar populatsiyasi taqriban 28 million ekanligini ko‘rsak bo‘ladi.

Hosil bo‘lgan egri chiziq shakli ko‘rsatkichli funksiya grafigiga o‘hshashligi ko‘rinib turibdi. Bu funksiyani $b(t)$ deb belgilab, (bu yerda $t\geq 0$), yoza olamiz: $b(t)=20\cdot2^{t-1} =10\cdot2^t$.▲

Umumiy holda, $b(t)=b_0a^t$ qonuniyat bilan o‘zgaradigan miqdor (bu yerda $b_0>0,\ a>1,\ t\geq0$) eksponentsial o‘suvchi miqdor deyiladi.

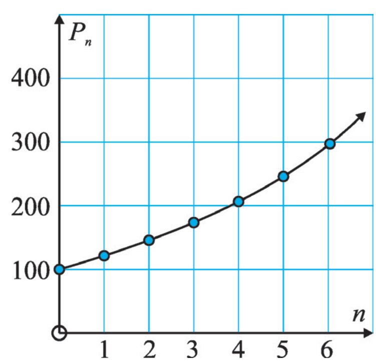
Quyidagi xulosaga ega bo‘lamiz:

Agar populatsiyaning miqdoriy o‘sishi uning boshlang‘ich (dastlabki) soniga proporsional bo‘lsa, bunday populatsiya eksponensial ko‘payadi.

“Eksponensial o‘sish” iborasi odatda qandaydir shiddatli, to‘xtovsiz o‘sish jarayonini ifodalaydi. Masalan, jonzotlar populatsiyasi, biror mamlakat aholisining shiddatli o‘sishini matbuotda shunday ta’riflashadi.

2- misol. Epidemiologiya hizmatining ma’lumotiga ko‘ra, sichqonlar populatsiyasi miqdori qulay sharoitlarda har haftada 20% ortar ekan. Dastlab 100 ta sichqon bo‘lsa, ularning populatsiyasi miqdori qanday qonuniyat bilan o‘sishini toping.

△Agar P_n deb n haftada davomidagi populatsiya miqdorini belgilasak, quyidagilarga ega bo‘lamiz: $P_0=100$ (dastlabki miqdor), $P_1=P_0\cdot1{,}2=100\cdot1{,}2$, $P_2=P_1\cdot1{,}2=100\cdot(1{,}2)^2$, $P_3=P_2\cdot1{,}2=100\cdot(1{,}2)^3$, va h.k. n haftada populatsiya miqdori $P_n=100\cdot(1{,}2)^n$ bo‘ladi.▲



Kalkulatordan foydalanib, mos qiymatlarni hisoblasak, quyidagi grafikka ega bo‘lamiz:

Ko‘rinib turibdiki, 6 haftada populatsiya miqdori qariyb 3 marta ortar ekan.

3- misol. Entomolog olim chigirtkalar populatsiyasining qishloq xo‘jaligi dalalariga zarar yetkazishini o'rganganda zarar ko'rgan maydonlar yuzi $A_n=1000\cdot 2^{0,2n}$ (gektar) qonuniyat bilan o'zgarishini aniqladi, bu yerda n haftalar soni.

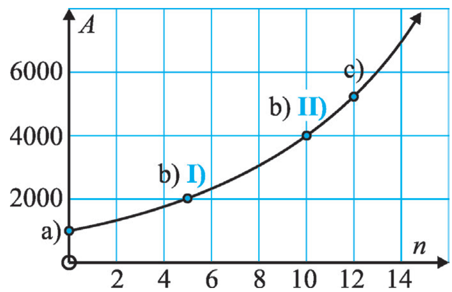
a) Dastlab qanday maydonga zarar etkazilgan?

b) **I)** 5, **II)** 10 haftada qanday maydonga zarar yetkaziladi?

c) Kalkulatordan foydalanib, 12 haftada qanday maydonga zarar yetkazilishini toping.

d) Zarar ko'rgan maydon yuzining haftalar soniga bog'lanish qonuniyatining grafigini chizing.

△ a) $A_0=1000\cdot 2^{0,2\cdot 0}=1000$ (gektar). Demak, dastlab 1000 ga maydonga ziyon yetkazilgan.



b) **I)** $A_3=1000\cdot 2^{0,2\cdot 5}=2000$ zarar ko'rgan maydon yuzi 2000 (ga) ga teng.

II) $A_{10}=1000\cdot 2^{0,2\cdot 10}=4000$ zarar ko'rgan maydon yuzi 4000 (ga) ga teng.

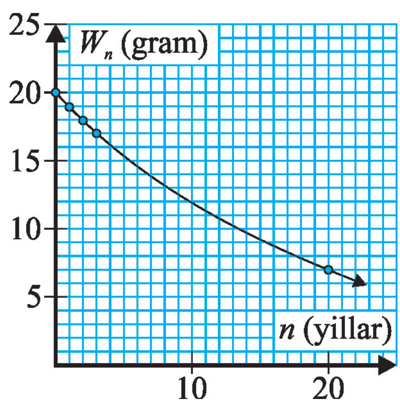
c) $A_{12}=1000\cdot 2^{0,2\cdot 12}=1000\cdot 2^{2,4}\approx 5280$ zarar ko'rgan maydon yuzi taqriban 5280 gektarga teng. ▲

4- misol. Radioaktiv yemirilish natijasida massasi 20 gramm bo'lgan radioaktiv modda har yili 5% ga kamayadi. W_n deb moddaning n yildagi massasini belgilasak,

$W_0=20$ g;
$W_1=W_0\cdot 0{,}95=20\cdot 0{,}95$ g;
$W_2=W_1\cdot 0{,}95=20\cdot (0{,}95)^2$ g;
$W_3=W_2\cdot 0{,}95=20\cdot (0{,}95)^3$ g;
$W_{20}=20\cdot (0{,}95)^{20}\approx 7{,}2$ g;
$W_{100}=20\cdot (0{,}95)^{100}\approx 0{,}1$ g

tengliklarga ega bo'lamiz.

Bundan $W_n=20\cdot (0{,}95)^n$.



$b(t)=b_0a^t$ qonuniyat bilan o'zgaradigan miqdor (bu yerda $b_0>0$, $0<a<1$, $t\geq 0$) eksponentsial kamayuvchi miqdor deyiladi.

5- misol. Iste'mol qilingan dori inson tanasiga asta-sekin singib, uning t soatdan so'ng qolayotgan miqdori (dozasi) $D(t)=120\cdot (0{,}9)^t$ (mg) qonuniyat bilan o'zgaradi.

a) $t=0$, 4, 12, 24 soat bo'lganda $D(t)$ ni toping.

b) Dastlab inson tanasiga qanday doza kiritilgan?

c) *a)* dagi ma’lumotlardan foydalanib, $D(t)$ grafigini tasvirlang, bu yerda $t \geq 0$.

d) Grafikdan foydalanib, 25 mg miqdordagi dori inson tanasida qancha vaqt qolishini baholang.

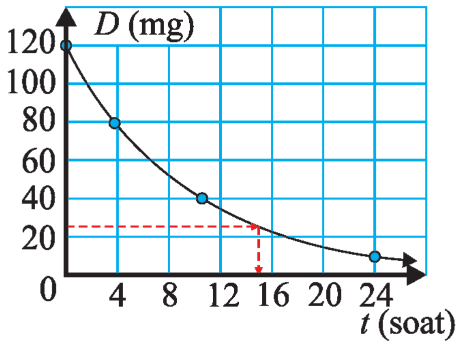
△ a) $D(t)=120\cdot(0{,}9)^t$ mg

$D(0)=120\cdot(0{,}9)^0=120$ mg; $D(4)=120\cdot(0{,}9)^4\approx 78{,}7$ mg;

$D(12)=120\cdot(0{,}9)^{12}\approx 33{,}9$ mg; $D(24)=120\cdot(0{,}9)^{24}\approx 9{,}57$ mg;

b) $D(0)=120$ bo‘lgani uchun, dastlab 120 (mg) dori kiritilgan.

c)



Shu grafikdan foydalanib, inson tanasiga kiritilgan 120mg dorining taxminan 15 soatdan so‘ng 25 mgi qolishini aniqlaymiz. ▲

6- misol. Radioaktiv yemirilish natijasida radioaktiv modda massasi $W_t=W_0\cdot 2^{-0{,}001t}$ gramm qonuniyat bo‘yicha o‘zgaradi, bu erda t yillar.

a) Dastlab modda qanday massaga ega bo‘lgan?

b) 200 yildan so‘ng moddaning necha foizi qoladi?

△ $t=0$ bo‘lganda $W_t=W_0\cdot 2^0=W_0$ bo‘ladi. Demak, moddaning dastlabki massasi W_0 ekan. $t=200$ bo‘lganda $W_{200}=W_0\cdot 2^{-0{,}001\cdot 200}=W_0\cdot 2^{-0{,}2}\approx W_0\cdot 0{,}8706$. Demak, 200 yildan so‘ng moddaning taxminan 87,1 foizi qoladi. ▲

7- misol. Dengiz sathidan h km balandlikka ko‘tarilganimizda, atmosfera bosimi $p=76\cdot 2{,}7^{-\frac{h}{8}}$ (sm simob ustuni) qonuniyat bilan o‘zgarar ekan. 5,6 km balandlikda atmosfera bosimi qanday bo‘ladi?

8- misol. Dengiz sathidan balandlik $h=\dfrac{8000}{0{,}4343}\lg\dfrac{p_0}{p}$ formula bilan hisoblanadi, bu yerda $p_0=760$ mm simob ustuni – dengiz sathidagi atmosfera bosimi, p esa h (m) balandlikdagi atmosfera bosimi. Alpinistlar toqqa ko‘tarilganda 304 mm simob ustuni bosim bo‘lganini aniqlashdi. Alpinistlar qanday balandlikka ko‘tarilishdi?

$$h=\frac{8000}{0{,}4343}\lg\frac{760}{304}\approx 7330{,}2 \text{ m}.$$

9- misol. Radioaktiv modda massasi vaqt o‘tish bilan $m(t)=m_0\cdot\left(\frac{1}{2}\right)^{\frac{t}{T}}$ qonuniyatga ko‘ra kamayadi, bu yerda m_0 ‘ boshlang‘ich vaqtdagi massa, $m-t$ vaqtdagi massa, T – radioaktiv yemirilish tezligini ifodalovchi koeffitsiyent (yarim yemirilish davri).

Hozirgi kunda saqlanib qolgan moddaning m massasini bilsak, necha yilda massa m_0 dan m gacha kamayganini topa olamiz:

$$t=-T\log_2\left(\frac{m(t)}{m_0}\right).$$

Bunday munosabat tarixiy tadqiqotlarda ham qo‘llanilishini aytish joiz.

10- misol. Tabiiy til lug‘atidagi so‘zlar soni vaqt o‘tishi bilan $N(t)=N_0\cdot e^{-\lambda t}$ qonuniyat bilan kamayishi kuzatilgan, bu yerda N_0 – boshlang‘ich vaqtdagi so‘zlar soni, $N(t)-t$ (ming yillar) vaqtdagi saqlanib qolgan so‘zlar soni, λ – tildagi so‘zlarning saqlanib qolishini ifodalovchi koeffitsiyent.

Hozirgi kunda saqlanib qolgan so‘zlar $N(t)$ miqdorini bilsak, necha yilda so‘zlar hajmi N_0 dan $N(t)$ gacha kamayganini topa olamiz:

$$t=-\frac{1}{\lambda}\cdot\ln\left(\frac{N(t)}{N_0}\right).$$

11- masala. Dastlab shahar aholisi a kishi bo‘lib, aholi soni har yili 10 % ga ortsa, aholining x yildan keyin qancha bo‘lishini aniqlovchi formulani toping.

△ Murakkab foiz formulasiga ko‘ra, shahar aholisi soni x yildan so‘ng $y=a\cdot\left(\frac{100+10}{100}\right)^x=a\cdot(1{,}1)^x$ bo‘ladi: Demak, $y=a\cdot(1{,}1)^x$ formula yordamida a berilganda x yildan so‘ng aholi soni qancha bo‘lishini aniqlash mumkin bo‘ladi. $a=1000000$ va yillar soni x bo‘yicha aholi sonini aniqlovchi jadvalni keltiramiz:

x	y	x	y
1	1 100 000	11	2 853 117
2	1 210 000	12	3 138 428
3	1 331 000	13	3 452 271
4	1 464 100	14	3 797 498
5	1 610 510	15	4 177 248
6	1 771 561	16	4 594 973
7	1 948 717	17	5 054 470
8	2 143 589	18	5 559 917
9	2 357 948	19	6 115 909
10	2 593 742	20	6 727 500

Jadvalga ko‘ra aholi soni 5 yildan so‘ng 1 610 510, 10 yildan so‘ng 2 593 742, 20 yildan so‘ng 6 727 500 kishi bo‘lar ekan. ▲

12- masala. Dastlab shahar aholisi a kishi bo‘lib, aholi soni har yili 2 % ga kamaysa, aholining x yildan keyin qancha bo‘lishini aniqlovchi formulani toping.

△ Murakkab foiz formulasiga ko‘ra shahar aholisi soni x yildan so‘ng $y = a \cdot \left(\frac{100-2}{100}\right)^x = a \cdot 0{,}98^x$ bo‘ladi. Demak, $y = a \cdot 0{,}98^x$ formula yordamida a berilganda x yildan so‘ng aholi sonini aniqlash mumkin. a=2000000 va yillar soni x bo‘yicha aholi sonini aniqlovchi jadvalini keltiramiz:

Jadvalga ko‘ra aholi soni 5 yildan so‘ng 1 807 842, 10 yildan so‘ng 1 634 146, 20 yildan so‘ng 1 335 216 kishi bo‘lar ekan. ▲

x	y	x	y
1	1 960 000	11	1 601 463
2	1 920 800	12	1 569 433
3	1 882 384	13	1 538 045
4	1 844 736	14	1 507 284
5	1 807 842	15	1 477 138
6	1 771 685	16	1 447 595
7	1 736 251	17	1 418 644
8	1 701 526	18	1 390 271
9	1 667 496	19	1 362 465
10	1 634 146	20	1 335 216

13- masala. Shahar aholisi dastlab a kishi edi. Agar aholi soni har yili 10 % ga ortsa, aholining x yildan keyin qancha bo‘lishini va necha yildan keyin k marta ortishini aniqlovchi formulani toping.

△ Ma’lumki, $y = a \cdot 1{,}1^x$ va masala shartidan $y = k \cdot a$ ekanini hisobga olib $k = 1{,}1^x$ yoki $x = \log_{1{,}1} k$ formula topiladi. Quyida aholi soni k marta ortishi uchun kerakli yillar sonini aniqlovchi jadval keltirilgan:

k	y	k	y	k	y
1	0	6	19	11	25
2	7	7	20	12	26
3	12	8	22	13	27
4	15	9	23	14	28
5	17	10	24	15	28

Jadvaldan ma’lumki, aholi soni 2 marta ortishi uchun 7 yil;

5 marta ortishi uchun 17 yil;

10 marta ortishi uchun 24 yil kerak ekan. ▲

14- masala. Shahar aholisi har yili 2 % ga kamaysa hamda aholining boshlang‘ich soni a nafar bo‘lsa, aholining x yildan keyin qancha bo‘lishini va necha yildan keyin k marta kamayishini aniqlovchi formulani toping.

△ Ma’lumki, $y=a\cdot 0{,}98^x$ va masala shartidan $y=\frac{a}{k}$ bo‘lishini inobatga olib $1/k=0{,}98^x$ yoki $x=\log_{0{,}98}(1/k)$ formula topiladi. Quyida aholi soni k marta kamayishi uchun kerakli yillar sonini aniqlovchi jadval keltirilgan:

k	$1/k$	x	k	$1/k$	x
1	1	0	11	0,090909	119
2	0,5	34	12	0,083333	123
3	0,333333	54	13	0,076923	127
4	0,25	69	14	0,071429	131
5	0,2	80	15	0,066667	134
6	0,166667	89	16	0,0625	137
7	0,142857	96	17	0,058824	140
8	0,125	103	18	0,055556	143
9	0,111111	109	19	0,052632	146
10	0,1	114	20	0,05	148

Jadvaldan ma’lumki, aholi soni: 2 marta kamayishi uchun 34 yil;

5 marta kamayishi uchun 80 yil;

10 marta kamayishi uchun 114 yil kerak ekan. ▲

15- masala. 1935- yili amerikalik seysmolog Ch. Rixter zilzilalarni tasniflash uchun 1 – 9,5 ballik magnitudalar shkalasini taklif qilgan. Bunda zilzila vaqtida yuzga keluvchi seysmik to‘lqin energiyasi intensivlik deb nomlanuvchi kattalik orqali baholandi. Rixter shkalasida intensivligi I bo‘lgan zilzilaning R magnitudasi $R=\lg I$ formula yordamida topilar ekan.

1966- yili Toshkentda 5,2 magnitudali, 2010- yili Gaitida esa 7 magnitudali zilzila ro‘y bergan. Shu zilzilalarni intensivlik bo‘yicha solishtiraylik.

△ Gaiti zilzilasi: $7=\lg I_1$, bundan $I_1=10^7=10\,000\,000$;

Toshkent zilzilasi: $5{,}2=\lg I_2$, bundan $I_2=10^{5,2}\approx 158\,489{,}3$;

Bundan $\frac{I_1}{I_2}\approx 63{,}1$. Demak, Gaitida Toshkentdagiga nisbatan taxminan 63 marta kuchliroq zilzila ro‘y bergan. ▲

Savol va topshiriqlar

1. Ko‘rsatkichli modelga misol keltiring.
2. Logarifmik modelga misol keltiring.

Mashqlar

192. Tomorqaga ishlov berilmasa, t kundan so‘ng begona o‘tlar yuzi $A(t)=3\cdot2^{0,1t}$ (kv. m) bo‘lgan yer maydonini qoplab, foydali o‘simliklarga ziyon yetkazadi.
a) Dastlab qancha maydonga ziyon yetkazilgan?
b) **I)** 2, **II)** 10, **III)** 30 kunda qanday maydonga ziyon yetkaziladi?
c) *a), b)* da olingan ma’lumotlardan foydalanib, ziyon ko‘rgan maydon yuzining kunlar soniga bog‘lanish qonuniyati grafigini chizing.

193. Orolbo‘yi ekologik tizimini tiklash maqsadida noyob hayvonlar populatsiyasini ko‘paytirish loyihasida ekologlar 25 ta juftlik hayvonlarni ko‘paytirmoqchi. Tadqiqotlarga ko‘ra, berilgan sharoitlarda bu hayvonlar populatsiyasi miqdori $P_n=P_0\cdot1{,}23^n$ qonuniyat bilan o‘zgaradi, bu yerda P_n – n yildagi hayvonlar soni.
a) P_0 soni nimani bildiradi?
b) **I)** 2, **II)** 5, **III)** 10 yilda qanday populatsiyaga ega bo‘lamiz?
c) *a), b)* da olingan ma’lumotlardan foydalanib, populatsiya miqdorining yillar soniga bog‘lanish qonuniyatining grafigini chizing.

194. Kimyoviy reaktsiya tezligi $V_t=V_0\cdot2^{0,05t}$ qonuniyat bilan o‘zgarar ekan, bu yerda t(°C) – temperatura.
a) 0 °C, b) 20 °C temperaturada reaktsiya tezligi qanday bo‘ladi?
c) 20 °C temperaturadagi reaktsiya tezligi 0 °C temperaturadagi reaksiya tezligiga nisbatan necha foiz ortadi?
d) $\left(\dfrac{V_{50}-V_{20}}{V_{20}}\right)\cdot100\%$ qiymatni hisoblang va ma’nosini tushuntiring.

195. 2017- yili Alyaska yarimoroli yonidagi orolga ayiqlarning 6 ta juftligi qo‘yib yuborildi. Dastlab orolda ayiqlar yo‘q edi. Ayiqlar populatsiyasi $B_t=B_0\cdot2^{0,18t}$ qonuniyat (bu yerda t - yillar) bilan o‘zgarsa, hisoblash vositalardan foydalanib, quyidagilarga javob bering:
a) B_0 soni nimani bildiradi? U nechaga teng?
b) 2037- yilda qanday populatsiyaga ega bo‘lamiz?
c) 2037- yildagi ayiqlar soni 2027- yildagi ayiqlar soniga nisbatan necha foiz ortadi?

196. Radioaktiv yemirilish natijasida radioaktiv modda massasi $W(t)=250\cdot(0{,}998)^t$ (g) qonuniyat bo‘yicha o‘zgaradi, bu yerda t – yillar.
a) Dastlab modda qanday massaga ega bo‘lgan?
b) **I)** 400, **II)** 800, **III)** 1200 yilda moddaning necha grammi qoladi?
c) Yuqoridagi ma’lumotlardan foydalanib, $W(t)$ ning grafigini tasvirlang.
d) Grafikdan foydalanib, modda qachon 125 mg miqdorda qolishini baholang.

197. Qaynoq suv sovitilganda uning T temperaturasi $T(t)=100\cdot2^{-0{,}02t}$ °C qonuniyat bilan o‘zgarar ekan, bu yerda t - minutlar.
a) Dastlab qanday temperatura bo‘lgan?
b) **I)** 15; **II)** 20 minutdan keyin temperatura nechaga teng bo‘ladi?
c) Yuqoridagi ma’lumotlardan foydalanib, $W(t)$ ning grafigini tasvirlang.
d) Grafikdan foydalanib, 78 minutdan keyin temperatura nechaga teng bo‘lishini baholang.

198. Elektr zanjirdagi tok kuchi $I_t=0{,}6\cdot2^{-5t}$ (A) qonuniyat bilan o‘zgarar ekan, bu yerda t – sekundlar.
a) Dastlab qanday tok kuchi bo‘lgan?
b) **I)** 0,1; **II)** 0,5; **III)** 1 sekunddan keyin tok kuchi nechaga teng bo‘ladi?
c) Yuqoridagi ma’lumotlardan foydalanib, $W(t)$ ning grafigini tasvirlang.

199. Dengizda d metr chuqurlikka nisbatan yorug‘lik $L(d)=L_0\cdot(0{,}9954)^d$ (kandela) qonuniyat bilan o‘zgarar ekan.
a) Dengiz tubida qanday yorug‘lik bo‘lgan?
b) 1000 metr chuqurlikdagi yorug‘lik necha foizga kamayadi?

200. 8 ta bakteriya populatsiyasi 2 soatdan so‘ng 100 tagacha o‘sdi. Shu sharoitlarda qachon populatsiya 500 taga etadi?

201. Uyali aloqa kompaniyasi ma’lumotlariga ko‘ra, kompaniya uyali aloqasidan foydalanuvchilar soni $N(t)=100000e^{0{,}09t}$ formula yordamida ifodalanar ekan, bu yerda t – oylar. Xozirgi kunda 3 mln foydalanuvchilar borligi ma’lum bo‘lsa, kompaniya qachon ish boshlagan?

202. Ovqat mikroto‘lqinli pechdan olinganda, u $T(t)=80e^{-0{,}12t}$ qonuniyatga asosan soviydi, bu yerda t – minutlar. Hozir xona temperaturasi 22°C bo‘lsa, necha minutdan so‘ng ovqat shu temperaturagacha soviydi?

203. Sun’iy yo‘ldosh balandligi t (yillar) vaqt o‘tishi bilan $H(t)=30000e^{-0{,}2t}$ qonuniyat bilan o‘zgarar ekan.
a) 2 yildan so‘ng balandlik qanday bo‘lishini hisoblang.
b) Yo‘ldosh 320 km balandlikda bo‘lsa, u atmosferaning yuqori qatlamlarida yonib ketadi. Shu paytgacha qancha vaqt o‘tadi?

III BOBGA DOIR MASHQLAR

Tenglamalarni yeching (**204–205**):

204. a) $x^4-1=0$; b) $5x^4-3x^3-4x^2-3x+5=0$; c) $3x^4-4x^3-7x^2-4x+5=0$.

205. a) $(x-3)(x+14)(x-15)=0$; b) $(4x+11)(3x-5)=0$; c) $x^4-15x^2-16=0$; d) $x^4+24x^2-25=0$.

Tengsizliklarni yeching (**206–208**):

206. a) $(2-x)(3x+1)(2x-3)>0$; b) $(3x-2)(x-3)^3(x+1)^3(x+2)^4>0$.

207. a) $x^4+8x^3+12x^2\geq 0$; b) $(16-x^2)(x^2+4)(x^2+x+1)(x^2-x-x)\leq 0$.

208. a) $\dfrac{x^4-2x^2-8}{x^2+x-1}<0$; b) $\dfrac{3x-2}{2x-3}<3$; c) $\dfrac{7x-4}{x+2}\geq 1$; d) $\dfrac{1}{x+1}+\dfrac{2}{x+3}<\dfrac{3}{x+2}$.

209. Tenglamalar sistemasini yeching:

a) $\begin{cases} x^2+y^2=113, \\ xy=56; \end{cases}$ b) $\begin{cases} x^2y+xy^2=84, \\ x^3+y^3=91; \end{cases}$

c) $\begin{cases} x^2+9xy+2y^2=12, \\ 2x^2+3xy-4y^2=1; \end{cases}$ d) $\begin{cases} x^2-2xy+3y^2=2, \\ x^2+xy+y^2=3. \end{cases}$

Tengsizliklar sistemasini yeching (**210–211**):

210. a) $\begin{cases} \dfrac{3x+5}{7}+\dfrac{10-3x}{5}>\dfrac{2x+7}{3}-7\dfrac{3}{21}, \\ \dfrac{7x}{3}-\dfrac{11(x+1)}{6}>\dfrac{3x-1}{3}-\dfrac{13-x}{2}; \end{cases}$ b) $\begin{cases} \dfrac{2x-11}{4}+\dfrac{19-2x}{2}<2x, \\ \dfrac{2x+15}{9}>\dfrac{x-1}{5}+\dfrac{x}{3}. \end{cases}$

211. a) $\begin{cases} 2x^2+2<5x, \\ x^2\geq x; \end{cases}$ b) $\begin{cases} 2x^2+2<5x, \\ x^2\geq x; \end{cases}$ c) $\begin{cases} \dfrac{(x+2)(x^2-3x+8)}{x^2-9}\leq 0, \\ \dfrac{1-x^2}{x^2+2x-8}\geq 0. \end{cases}$

212. Irratsional tenglamani yeching:

a) $\sqrt{8x+1}+\sqrt{3x-5}=\sqrt{7x+4}+\sqrt{2x-2}$;

b) $\sqrt{2x+3}+\sqrt{3x+2}-\sqrt{2x+5}=\sqrt{3x}$;

c) $\dfrac{\sqrt{3+2x}}{2x^2-x-1}>0$; d) $\sqrt{x-2}-\sqrt{x-3}>-\sqrt{x-5}$.

Sonlarni taqqoslang (**213–215**):

213. a) $4{,}2^{-\sqrt{2}}$ va 1; b) $0{,}2^{\frac{3}{5}}$ va $0{,}2^{-\frac{3}{5}}$; c) $(0{,}4)^{-\frac{\sqrt{5}}{2}}$ va 1.

214. a) $4^{0,5}$ va $4^{\frac{\sqrt{3}}{3}}$; b) $\sqrt{3}^{\,0,2}$ va $3^{0,2}$; c) $2^{-\frac{3}{4}}$ va $8^{-\frac{4}{9}}$.

215. a) $2^{-\sqrt{3}}$ va $2^{-\sqrt{5}}$; b) $7^{-0,3}$ va $7^{-\frac{1}{3}}$; c) $(\frac{1}{3})^{\sqrt{5}}$ va $3^{-\sqrt{3}}$.

216. Funksiyaning aniqlanish sohasini toping:

a) $y=5^{\sqrt{x^2-1}}$; b) $y=\dfrac{1}{3^x+1}$; c) $y=\dfrac{1}{3^{x^2}-9}$; d) $y=3^{\frac{1}{2-x}}$.

217. Funksiyaning qiymatlar sohasini toping:

a) $y=2^{-|x|}$; b) $y=3+4^{x+1}$; c) $y=-6^x$; d) $y=5^{|x|}+1$.

Tenglamalarni yeching (**218–219**):

218. a) $8^x=2^{\frac{1}{5}}$; | b) $121^x-7\cdot 11^x=5\cdot 11^x-11$; | c) $0,5^{\,x^2+x-3,5}=2\sqrt{2}$.

219. a) $6^{2x}-5^{2x-1}=6^{2x-1}+5^{2x}$; | b) $4^{x+3}+4^x=130$; | c) $125^x+20^x=2^{3x+1}$.

220. Tenglamalar sistemasini yeching:

a) $\begin{cases} x+y=5, \\ 5^{\,y-x^2}=0,2; \end{cases}$ b) $\begin{cases} 3^{x-1}=2^y, \\ 0,1^{2x-y}=0,01; \end{cases}$ c) $\begin{cases} 5^{x-y}=25, \\ 3^{x+y}=27. \end{cases}$

221. Tengsizlikni yeching:

a) $4^x\le 3^x$; | b) $16^x-7\cdot 4^x-8<0$; | c) $4^x\cdot 5^{1-x}<\dfrac{25}{4}$; | d) $6^{\frac{x-3}{x+8}}\ge 1$.

222. Sonlarni taqqoslang:

a) $\log_3 2$ va 2; b) $\log_3 5$ va $2\cdot\log_3 2$; c) $\log_2 5$ va $\log_5 2$

e) $\log_{0,2} 5$ va $\log_{0,2} 6$; d) $\log_4 3$ va $\log_3 4$; f) $\lg 18,8$ va $\lg 6\pi$.

223. Funksiyaning aniqlanish sohasini toping:

a) $y=\log_2(2x+7)$; | b) $y=\log_{\frac{1}{3}}(4-x^2)$; | c) $y=\log_5(-8x)$; | d) $y=\lg\dfrac{x-3}{x+8}$.

Tenglamalarni yeching:

224. a) $\lg(x-9)+\lg(2x-1)=2$; b) $\log_2\sqrt{x-3}+\log_2\sqrt{x+3}=2$.

Tenglamalar sistemasini yeching (**225–226**):

225. a) $\begin{cases} 5^{x-y}=1, \\ 2^{\log_2(x+y)}=6; \end{cases}$ b) $\begin{cases} \lg x+\lg y=4, \\ \lg x-\lg y=6; \end{cases}$ c) $\begin{cases} \log_{17}(3^x+2^y)=1, \\ 3^{x+1}-4\cdot 2^y=-5; \end{cases}$

226. a) $\begin{cases} 2^x\cdot 5^y=40, \\ 5^x\cdot 2^y=250; \end{cases}$ b) $\begin{cases} \log_2 x+5^{\log_5 y}=4, \\ x^y=16; \end{cases}$ c) $\begin{cases} 3^x\cdot 3^y=81, \\ 3^x-3^y=24. \end{cases}$

227. Tengsizlikni yeching:

a) $\log_3(x^2+x+1)\geq 1$; b) $\log_2(x^2+x-6)-\log_2(x+3)\leq 1$;

c) $\lg^2 x<\lg x^5-6$; d) $\log_3(4^x-5\cdot 2^x+13)>2$; e) $5^{x+7}>2$.

228. Funksiya grafigini chizing:

a) $y=1,5\sin(2x-1)$; b) $y=2\cos(2x-\frac{\pi}{3})$; c) $y=\log_3(1-x)$.

229. Taqqoslang:

a) $\arcsin(-\frac{1}{2})$ va $\arccos\frac{\sqrt{3}}{2}$; b) $\arccos\frac{1}{2}$ va $\mathrm{arctg}(-1)$;

c) $\mathrm{arc\,tg}\sqrt{3}$ va $\mathrm{arctg}1$; d) $\arccos\left(-\frac{1}{2}\right)$ va $\arcsin\left(-\frac{\sqrt{3}}{2}\right)$.

230. Hisoblang:

a) $2\arcsin(-\frac{\sqrt{3}}{2})+\mathrm{arctg}(-1)+\arccos\frac{\sqrt{2}}{2}$;

b) $\mathrm{arctg}(-\sqrt{3})+\arccos\left(-\frac{\sqrt{3}}{2}\right)+\arcsin 1$;

Tenglamani yeching (**231–233**):

231. a) $2\cos^2x+5\sin x-4=0$; b) $3\sin^2 2x+7\cos^2x-3=0$; c) $4\mathrm{tg}^2x-5\mathrm{tg}x+1=0$.

232. a) $3\sin^2x+7\sin x-10=0$; b) $2\cos^2x-5\cos x+3=0$; c) $\sin 6x=\sin 3x$.

233. a) $\cos 7x=\cos 2x$; b) $\mathrm{tg}8x=\mathrm{tg}11x$.

Tengsizlikni yeching (**234–235**):

234. a) $\sin x>-\frac{1}{2}$; b) $\cos 2x\leq\frac{1}{2}$; c) $\mathrm{tg}3x\geq 1$; d) $\sin 2x\leq\frac{1}{2}$.

235. a) $\sin 4x\leq\frac{1}{2}$; b) $\cos 10x\geq 0$; c) $\mathrm{tg}9x\leq\sqrt{3}$; d) $\cos\left(2x-\frac{\pi}{4}\right)\leq 0$.

Nazorat test topshiriqlari

1. Tenglamani yeching: $\sin 6x=0$.

A) $x=\frac{\pi}{6}n,\ n\in Z$; *B*) $x=\frac{\pi}{5}n,\ n\in Z$;

C) $x=\frac{\pi}{4}n\ \ n\in Z$ *D*) $x=\frac{\pi}{3}n,\ n\in Z$.

2. Tenglamani yeching: $\cos 2x=0$.

$A)\ x=2\pi n,\ n\in Z;$ $B)\ x=\pi n,\ n\in Z;$

$C)\ x=\frac{\pi}{4}+\frac{\pi}{2}n,\ n\in Z;$ $D)\ x=\frac{\pi}{3}n,\ n\in Z.$

3. Tenglamani yeching: $\mathrm{tg}4x=\sqrt{3}$.

$A)\ x=\frac{\pi}{3}+\frac{\pi n}{4},\ n\in Z;$ $B)\ x=\frac{\pi}{2}+\frac{\pi n}{4},\ n\in Z;$

$C)\ x=\frac{\pi}{12}+\frac{\pi n}{4},\ n\in Z;$ $D)\ x=\frac{\pi n}{4},\ n\in Z.$

4. Tengsizlikni yeching: $\sin 2x>3$.

$A)\ x=\pi n,\ n\in Z;$ $B)\ \varnothing;$ $C)\ x=\frac{\pi}{2}+\pi n,\ n\in Z;$ $D)\ x=2\pi n,\ n\in Z.$

5. Tengsizlikni yeching: $\cos 2x<3$.

$A)\ (-\infty;+\infty);$ $B)\ \varnothing;$ $C)\ (-\infty;0);$ $D)\ (0;+\infty).$

6. Aniqlanish sohasini toping: $y=12^x$.

$A)\ (-\infty;+\infty);$ $B)\ (0;+\infty);$ $C)\ (-\infty;0);$ $D)\ \varnothing.$

7. Aniqlanish sohasini toping: $y=\log_2(3-x)$.

$A)\ (3;+\infty);$ $B)\ [3;+\infty);$ $C)\ (-\infty;3);$ $D)\ (-\infty;3].$

8. Hisoblang: $\arcsin\frac{1}{2}$.

$A)\ \frac{\pi}{2};$ $B)\ \pi;$ $C)\ \frac{\pi}{4};$ $D)\ \frac{\pi}{6}.$

9. Hisoblang: $\arccos\frac{\sqrt{3}}{2}$.

$A)\ \frac{\pi}{3};$ $B)\ \frac{\pi}{2};$ $C)\ \frac{\pi}{6};$ $D)\ \frac{\pi}{4}.$

10. Hisoblang: $\mathrm{arctg}1$.

$A)\ \frac{\pi}{3};$ $B)\ \frac{\pi}{2};$ $C)\ \frac{\pi}{6};$ $D)\ \frac{\pi}{4}.$

IV BOB

KOMPLEKS SONLAR

86-87 KOMPLEKS SONLAR VA ULAR USTIDA AMALLAR. KOMPLEKS SONNI TASVIRLASH

Kompleks sonlar

Kompleks sonlar haqidagi ta'limot ilm-u fanda, xususan, matematikada alohida o'rin tutadi. Tez rivojlanayotgan bu soha texnikada, shuningdek, ishlab chiqarishning ko'plab sohalarida g'oyat keng qo'llanishga ega. Shu sonlar haqida ayrim ma'lumotlarni keltiramiz. Xususiy bir misoldan boshlaylik.

$x^2+4=0$ tenglamani yechish jarayonida $x_1=2\sqrt{-1}$ va $x_2=-2\sqrt{-1}$ "sonlar" hosil bo'ladi. Haqiqiy sonlar orasida esa bunday "sonlar" mavjud emas. Sunday holatdan qutulish uchun $\sqrt{-1}$ ga son deb qarash zarurati paydo bo'ladi.

Bu yangi son hech qanday real kattalikning o'lchamini, yoki uning o'zgarishini ifodalamaydi. Shu sababli $\sqrt{-1}$ ni ***mavhum*** (hayoliy, haqiqatda mavjud bo'lmagan) ***birlik*** deb atash va maxsus belgilash qabul qilingan: $\sqrt{-1}=i$. Mavhum birlik uchun $i^2=-1$ tenglik o'rinlidir.

$a+bi$ ko'rinishdagi ifodani qaraymiz. Bu yerda a va b lar istalgan haqiqiy sonlar, i esa mavhum birlik.

$a+bi$ ifoda haqiqiy son a va mavhum son bi lar "kompleksi"dan iborat bo'lgani uchun uni kompleks son deb atash qabul qilingan.

$a+bi$ ifoda algebraik shakldagi kompleks son deb ataladi.

$a+bi$ ni "algebraik shakldagi kompleks son" deyish o'rniga qisqalik uchun "kompleks son" deb ataymiz. Kompleks sonlarni bitta harf bilan belgilash qulay. Masalan, $a+bi$ ni z bilan belgilaylik. $z=a+bi$ kompleks sonning haqiqiy qismi a ni Re(z) (fransuzcha réelle − haqiqiy) kabi, mavhum qismi b ni esa Im(z) (fransuzcha *imaginaire* – mavhum) kabi belgilash qabul qilingan: a=Re(z), b=Im(z).

Agar $z=a+bi$ kompleks son uchun $b=0$ bo'lsa, haqiqiy son $z=a$ hosil bo'ladi.

Demak, haqiqiy sonlar to‘plami R barcha kompleks sonlar to‘plami C ning qism to‘plami bo‘ladi: $R \subset C$.

1- misol. $z_1=1+2i$, $z_2=2-i$, $z_3=2{,}1$, $z_4=2i$, $z_5=0$ kompleks sonlarning haqiqiy va mavhum qismlarini toping.

△ Kompleks sonlarning haqiqiy va mavhum qismlarining ta’riflariga ko‘ra, topamiz:

$\mathrm{Re}(z_1)=1$; $\mathrm{Re}(z_2)=2$; $\mathrm{Re}(z_3)=2{,}1$; $\mathrm{Re}(z_4)=0$; $\mathrm{Re}(z_5)=0$;

$\mathrm{Im}(z_1)=2$; $\mathrm{Im}(z_2)=-1$; $\mathrm{Im}(z_3)=0$; $\mathrm{Im}(z_4)=2$; $\mathrm{Im}(z_5)=0$. ▲

Kompleks sonlar uchun “<”, “>” munosabatlari aniqlanmagan, lekin teng kompleks sonlar tushunchasi kiritiladi.

Haqiqiy va mavhum qismlari, mos ravishda, teng bo‘lgan kompleks sonlar o‘zaro teng kompleks sonlar deb ataladi.

Masalan, $z_1=1{,}5+\frac{4}{5}i$ va $z_2=\frac{3}{2}+0{,}8i$ sonlari uchun $\mathrm{Re}(z_1)=\mathrm{Re}(z_2)=1{,}5$; $\mathrm{Im}(z_1)=\mathrm{Im}(z_2)=0{,}8$. Demak, $z_1=z_2$.

Bir-biridan faqat mavhum qismlarining ishorasi bilan farq qiladigan ikki kompleks son o‘zaro qo‘shma kompleks sonlar deyiladi. $z=a+bi$ kompleks songa qo‘shma kompleks son $\bar{z}=a-bi$ ko‘rinishda yoziladi.

Masalan, $6+7i$ va $6-7i$ lar qo‘shma kompleks sonlardir: $\overline{6+7i}=6-7i$. Shu kabi $\bar{z}$ soniga qo‘shma son $\bar{\bar{z}}=z$ bo‘ladi. Masalan, $\overline{\overline{6+7i}}=\overline{6-7i}=6+7i$. a haqiqiy songa qo‘shma son a ning o‘ziga teng: $\bar{a}=\overline{a+0\cdot i}=a-0\cdot i=a$. Lekin, bi mavhum songa qo‘shma son $\overline{bi}=-bi$ dir. Chunki $\overline{bi}=\overline{0+bi}=0-bi=-bi$.

Kompleks sonlar ustida arifmetik amallar

Kompleks sonlar ustida arifmetik amallar quyidagicha aniqlanadi:

$$(a+bi)+(c+di)=(a+c)+(b+d)i; \qquad (1)$$

$$(a+bi)-(c+di)=(a-c)+(b-d)i; \qquad (2)$$

$$(a+bi)\cdot(c+di)=(ac-bd)+(ad+bc)i; \qquad (3)$$

$$\frac{a+bi}{c+di}=\frac{ac+bd}{c^2+d^2}+\frac{bc-ad}{c^2+d^2}i. \qquad (4)$$

(1) va (2) tengliklarni bevosita qo‘llash qiyin emas. Kompleks sonlarni ko‘paytirish amalini $i^2=-1$ ekanligini e'tiborga olib, ko‘phadlarni ko‘paytirish kabi bajarish mumkin.

2- misol. Yig‘indini toping: $(3+7i)+(-5+4i)$.

△ Yig‘indini topish uchun (1) formuladan foydalanamiz:

$(3+7i)+(-5+4i)=(3+(-5))+(7+4)i=-2+11i$. ▲

3- misol. Ayirmani toping: $(13-7i)-(-5+4i)$.

△Ayirmani topish uchun (2) formuladan foydalanamiz:
$(13-7i)-(-5+4i)=(13-(-5))+(-7-4)i=18-11i$. ▲

4- misol. Ko'paytmani toping: $(2-i)\cdot(\frac{3}{4}+2i)$.

△Ko'patmani topish uchun qavslarni ochamiz.

$$(2-i)\cdot\left(\frac{3}{4}+2i\right)=2\cdot\frac{3}{4}+2\cdot 2i-i\cdot\frac{3}{4}-2i^2=\frac{3}{2}+4i-\frac{3}{4}i+2=\frac{7}{2}+\frac{13}{4}i. \ ▲$$

$\dfrac{a+bi}{c+di}$ bo'linmani hisoblash uchun,uning surati va maxrajini maxrajning "qo'shmasi" $c-di$ ga ko'paytirib, tegishli amallarni bajarish lozim.

5- misol. Bo'lish amalini bajaring: $\dfrac{2-i}{-3+2i}$.

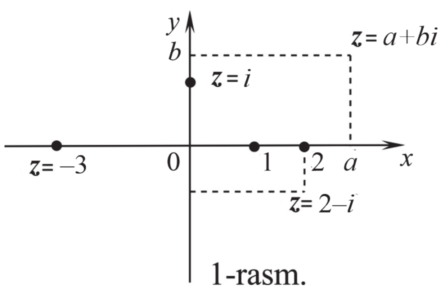
△ $$\frac{2-i}{-3+2i}=\frac{(2-i)(-3-2i)}{(-3+2i)(-3-2i)}=\frac{-6-4i+3i-2}{(-3)^2-(2i)^2}=\frac{-8-i}{13}=\frac{-8}{13}-\frac{1}{13}i. \ ▲$$

$z+w=0$ tenglikni qanoatlantiruvchi z, w kompleks sonlar o'zaro qarama-qarshi sonlar deyiladi. z kompleks soniga qarama-qarshi sonni $-z$ bilan belgilash qabul qilingan. $z=a+bi$ kompleks songa qarama-qarshi bo'lgan yagona kompleks son mavjud va bu son $-z=-a-bi$ kompleks sondan iborat.

$zw=1$ tenglikni qanoatlantiradigan z va w kompleks sonlar o'zaro teskari kompleks sonlar deyiladi. $z=0$ songa teskari son mavjud emas. Har qanday $z\neq0$ kompleks songa teskari kompleks son mavjud. Bu son $\dfrac{1}{z}$ sonidan iborat.

Kompleks sonni tekislikda tasvirlash

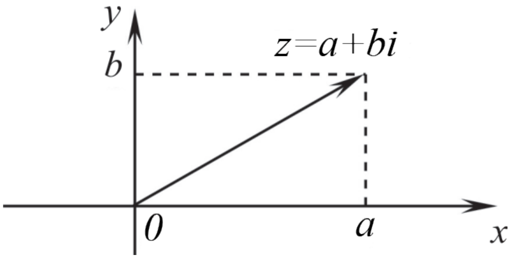
Faraz qilaylik, tekislikda to'g'ri burchakli. Dekart koordinatalar sistemasi berilgan bo'lsin. U holda $z=a+bi$ kompleks songa tekislikda koordinatalari $(a;\ b)$ bo'lgan nuqta mos keladi.



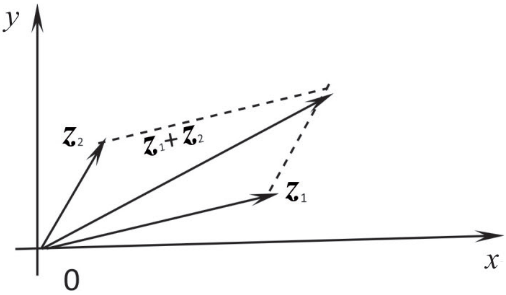
1-rasm.

Bu usul bilan tasvirlashda $a+0i$ kompleks songa $(a;\ 0)$ koordinatali nuqta, $0+bi$ kompleks songa esa $(0;\ b)$ nuqta mos keladi. Shung uchun ham x o'qiga haqiqiy o'q va y o'qiga mavhum o'q deyiladi (1- rasm).

$a+bi$ kompleks sonni tekislikda a va b koordinatali vektor kabi ham tasvirlash mumkin (2- rasm). Bu esa kompleks sonlarni qo'shishda vektorlarni qo'shishning parallelogram qoidasini qo'llash imkonini beradi (3- rasm).



2- rasm.



3- rasm.

Savol va topshiriqlar

1. Mavhum birlik nima? Nega uni kiritishga extiyoj sezildi?
2. Kompleks sonning algebraik ko‘rinishini yozing, misol keltiring
3. Ikkita ko‘mpleks son qachon teng deyiladi? Misol keltiring.
4. Ikkita ko‘mpleks sonning yig‘indisi, ayirmasi, ko‘paytmasi, bo‘linmasi qanday aniqlanadi? Misollarda tushuntiring.
5. Qarama-qarshi ko‘mpleks son nima?
6. Qo‘shma ko‘mpleks son nima?
7. O‘zaro teskari ko‘mpleks sonlar nima? Misollar keltiring.
8. Kompleks sonni vektor kabi tasvirlash nima? Misol keltiring.

Mashqlar

1. Kompleks sonlarning haqiqiy va mavhum qismlarini og‘zaki ayting:

1) $z=-3+7i$; 2) $z=4-\frac{1}{2}i$; 3) $z=-2-5i$;

4) $z=-5,7+5i$; 5) $z=5i$; 6) $z=9$;

7) $z=-7+3i$; 8) $z=8-\frac{1}{2}i$; 9) $z=-5-6i$;

10) $z=-5,7-5i$; 11) $z=-5i$; 12) $z=90$.

2. Kompleks sonlarni algebraik ko‘rinishda yozing:

1) $\mathrm{Re}(z)=4,\quad \mathrm{Im}(z)=-5$; 2) $\mathrm{Re}(z)=-2,\quad \mathrm{Im}(z)=3$;

3) $\mathrm{Re}(z)=0,\quad \mathrm{Im}(z)=8$; 4) $\mathrm{Re}(z)=7,\quad \mathrm{Im}(z)=0$;

5) $\mathrm{Re}(z)=6,\quad \mathrm{Im}(z)=-7$; 6) $\mathrm{Re}(3)=-3,\quad \mathrm{Im}(z)=4$;

7) $\mathrm{Re}(z)=0,\quad \mathrm{Im}(z)=9$; 8) $\mathrm{Re}(z)=2,\quad \mathrm{Im}(z)=0$;

9) $\mathrm{Re}(z)=12,\quad \mathrm{Im}(z)=20$.

3. Teng kompleks sonlarni ko‘rsating (**3–4**):

1) $2-4i$; | 2) $3+5i$; | 3) $\frac{2}{3}+i$; | 4) $\sqrt{121}-7i$; | 5) $33+44i$; | 6) $\sqrt[3]{8}+\sqrt[3]{27}\,i$.

4. 1) $4-3i$; | 2) $1+3i$; | 3) $\frac{1}{3}+i$; | 4) $\sqrt{16}-\sqrt{9}i$; | 5) $3+4i$; | 6) $\sqrt[3]{27}+\sqrt[3]{64}i$.

5. z soniga qo‘shma bo‘lgan $\bar{z}$ sonni toping (**5–6**):

1) $z=5-3i$; | 2) $z=-5+3i$; | 3) $z=1-i$; | 4) $z=2+3i$; | 5) $z=-7-i$.

6. 1) $z=7,2$; | 2) $z=6i$; | 3) $z=\sqrt{16}-\sqrt{9}i$; | 4) $z=-2i+(-7+3i)$.

7. Yig‘indini toping (**7–8**):

1) $(-5+3i)+(2-i)$; | 2) $(-3)+(3-4i)$; | 3) $(2+5i)+(-2-5i)$; | 4) $(-4i)+(3.6-3i)$.

8. 1) $(8-3i)+(8+3i)$; | 2) $(-7+5i)+(7-5i)$; | 3) $9i+(3-8i)$; | 4) $-17i+(-9+16i)$.

9. Ayirmani toping (**9–10**):

1) $(3+4i)-(4+2i)$; | 2) $(4-6i)-(3+2i)$; | 3) $(2+4i)-(-4+2i)$.

10. 1) $(5+4i)-(5-4i)$; | 2) $7-(8+5i)$; | 3) $7i-(6i+3)$.

11. Ko‘paytmani toping (**11–12**):

1) $(4+6i)(3+4i)$; | 2) $(5+8i)(3-2i)$; | 3) $(6-4i)(3-6i)$.

12. 1) $(-3+2i)(8-4i)$; | 2) $\left(\frac{1}{3}-i\right)\left(\frac{1}{2}+i\right)$; | 3) $\left(\frac{5}{7}+4i\right)\left(\frac{7}{5}-2i\right)$.

13. Bo‘linmani toping (**13–14**):

1) $\frac{2+2i}{1-2i}$; | 2) $\frac{4-5i}{3+2i}$; | 3) $\frac{3+4i}{3-4i}$; | 4) $\frac{2+3i}{4-3i}$; | 5) $\frac{4-5i}{3+2i}$.

14. 1) $\frac{4-5i}{-2+3i}$; | 2) $\frac{3}{5-2i}$; | 3) $\frac{5-2i}{3}$; | 4) $\frac{7i}{13-i}$; | 5) $\frac{7+4i}{5-6i}$.

15. Amallarni bajaring (**15–16**):

1) $\frac{(3-4i)(4-3i)}{2+i}$; | 2) $\frac{(4-i)(3+2i)}{3-2i}$; | 3) $\frac{5-2i}{(2+i)(1-i)}$.

16. 1) $\frac{3-2i}{(1+i)(3-i)}$; | 2) $\frac{3}{2-3i}+\frac{3}{2+3i}$; | 3) $\frac{2}{1+i}+\frac{5}{2+i}$.

17. Kompleks sonlarni tekislikda tasvirlang (**17–18**):

1) $z=3+4i$; | 2) $z=3-4i$; | 3) $z=-3+4i$; | 4) $z=-3-4i$; | 5) $z=2i$.

18. 1) $z=4-2i$; | 2) $z=5+3i$; | 3) $z=\frac{2+i}{2-i}$; | 4) $z=(2-i)(1+i)$; | 5) $z=(2+i)(2-i)$.

88 $r(\cos\varphi + i\sin\varphi)$ va $r\cdot e^{i\varphi}$ $(r>0,\ 0\le\varphi\le2\pi)$ KO'RINISHDAGI KOMPLEKS SONLAR

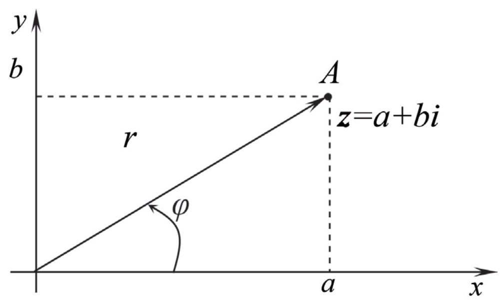
Bu mavzuda kompleks sonning trigonometrik va ko'rsatkichli ko'rinishlarini o'rganamiz.

Trigonometrik ko'rinishdagi kompleks sonlar

Tekislikda to'g'ri burchakli Dekart koordinatalar sistemasi berilgan bo'lsin. $z=a+bi$ kompleks songa $(a;\ b)$ koordinatali A nuqta mos qo'yilgan, deylik. Koordinatalar boshi O va A nuqtalarini tutashtirib $\overrightarrow{OA}$ vektorni hosil qilamiz (4-rasm).

O nuqtadan A nuqtagacha bo'lgan $r=OA$ masofa kompleks sonning moduli, abssissa o'qining musbat yo'nalishi hamda $\overrightarrow{OA}$ vektor orasidagi (φ) burchak ***kompleks sonning argumenti*** deyiladi.

Ravshanki, $0\le r<+\infty$, $0\le\varphi<2\pi$, $r=\sqrt{a^2+b^2}$, $\cos\varphi=\frac{a}{r}$, $\sin\varphi=\frac{b}{r}$.



4- rasm.

Kompleks sonning $z=r(\cos\varphi+i\sin\varphi)$ ko'rinishiga uning trigonometrik shakli va $z=r\cdot e^{i\varphi}$ ko'rinishiga esa ko'rsatkichli shakli deyiladi. Kompleks sonni trigonometrik ko'rinishidan algebraik ko'rinishiga o'tkazish uchun quyidagi formuladan foydalanadi: $a=r\cos\varphi$, $b=r\sin\varphi$.

1-misol. Kompleks sonlarni trigonometrik ko'rinishda yozing:

1) i; 2) $-2i$; 3) $-1-i$.

△ 1) $z=i=0+1\cdot i$, $a=0, b=1$, $r=\sqrt{0^2+1^2}=1$, $\cos\varphi=\frac{0}{1}=0$, $\varphi=\frac{\pi}{2}$.

Demak, $i=1\left(\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}\right)$, ya'ni $i=\cos\frac{\pi}{2}+i\sin\frac{\pi}{2}$.

2) $r=2$, $\varphi=\frac{3\pi}{2}$ bo'lganligi uchun $-2i=2\left(\cos\frac{3\pi}{2}+i\sin\frac{3\pi}{2}\right)$;

3) $z=-1-i$, $a=-1$, $b=-1$, $r=\sqrt{2}$, $\cos\varphi=-\frac{1}{\sqrt{2}}$, $\varphi=\frac{5\pi}{4}$.

Demak, $-1-i=\sqrt{2}\left(\cos\frac{5\pi}{4}+i\sin\frac{5\pi}{4}\right)$. ▲

2-misol. Kompleks sonlarni ko‘rsatkichli ko‘rinishda yozing:

1) i; 2) $-2i$; 3) $-1-i$.

△ 1-misolning hisoblashlaridan foydalanamiz:

$$i=e^{\frac{\pi}{2}i},\qquad -2i=2e^{\frac{3\pi}{2}i},\qquad -1-i=\sqrt{2}e^{\frac{5\pi}{4}i}.$$ ▲

Savol va topshiriqlar

1. Kompleks sonning moduli nima? U qanday hisoblanadi?
2. Kompleks sonning argumenti nima? Misol keltiring.
3. Kompleks sonning trigonometrik ko‘rinishini tushuntiring.
4. Kompleks sonning ko‘rsatkichli ko‘rinishini tushuntiring.
5. *Eylerning mashhur formulasini* isbotlang: $e^{i\pi}=-1$.

Mashqlar

Kompleks sonning modulini toping (**19–20**):

19. 1) $z=-2+3i$; 2) $z=-2+3i$; 3) $z=1+\sqrt{3}i$; 4) $z=\sqrt{8}-i$.

20. 1) $z=6-8i$; 2) $z=2+2\sqrt{3}i$; 3) $z=\sqrt{3}+i$; 4) $z=2i$.

Kompleks sonning argumentini toping (**21–22**):

21. 1) $z=\frac{\sqrt{3}}{2}+\frac{1}{2}i$; 2) $z=\frac{\sqrt{2}}{2}+\frac{\sqrt{2}}{2}i$; 3) $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$; 4) $z=2\sqrt{2}i$.

22. 1) $z=5$; 2) $z=-2i$; 3) $z=\frac{\sqrt{33}}{2}-\frac{\sqrt{11}}{2}i$.

Kompleks sonni trigonometrik va ko‘rsatkichli ko‘rinishda yozing (**23–24**):

23. 1) $z=-2-2i$; 2) $z=2-2i$; 3) $z=\sqrt{3}-i$; 4) $z=1-\sqrt{3}i$.

24. 1) $z=\sqrt{2}-\sqrt{2}i$; 2) $z=\frac{1}{2}-\frac{\sqrt{3}}{2}i$; 3) $z=\frac{\sqrt{33}}{2}-\frac{\sqrt{11}}{2}i$; 4) $z=\frac{\sqrt{3}}{2}+\frac{1}{2}i$.

89-90 TRIGONOMETRIK SHAKLDA BERILGAN KOMPLEKS SONLARNING KO‘PAYTMASI VA BO‘LINMASI

Trigonometrik shakldа berilgan kompleks sonlarni ko‘paytirish

$z_1=r_1(\cos\varphi_1+i\sin\varphi_1)$, $z_2=r_2(\cos\varphi_2+i\sin\varphi_2)$ trigonometrik korinishdagi kompleks sonlarning ko‘paytmasi uchun quyidagi formula o‘rinli:

$$z_1\cdot z_2=r_1\cdot r_2\left[\cos(\varphi_1+\varphi_2)+i\sin(\varphi_1+\varphi_2)\right]. \qquad (1)$$

z_1 va z_2 trigonometrik ko'rinishdagi sonlarni bo'lish uchun esa
$\frac{z_2}{z_1}=\frac{r_2}{r_1}\left[\cos\left(\varphi_2-\varphi_1\right)+i\sin\left(\varphi_2-\varphi_1\right)\right]$ formula o'rinli, $r_1 \neq 0$. (2)

1- misol. $z_1=3\left(\cos 20°+i\sin 20°\right)$ va $z_2=2\left(\cos 35°+i\sin 35°\right)$ kompleks sonlarni ko'paytiring.

Yuqoridagi qoidaga ko'ra, ko'paytmani topamiz:

$z_1\cdot z_2=3\cdot 2(\cos(20°+55°)+i\sin(20°+55°))=6\left(\cos 55°+i\sin 55°\right)$.

2- misol. $z_1=2\left(\cos 140°+i\sin 140°\right)$, $z_2=3\left(\cos 150°+i\sin 150°\right)$ va 5 cos
va $z_3=5\left(\cos 70°+i\sin 70°\right)$ kompleks sonlarni ko'paytiring.

Yuqoridagi qoidaga ko'ra ko'paytmani topamiz:

$$z_1\cdot z_2\cdot z_3=2\cdot 3\cdot 5\left[\cos\left(140°+150°+70°\right)+i\sin\left(140°+150°+70°\right)\right]=30\ \cos 360$$

$$=30\left(\cos 360°+i\sin 360°\right)=30.$$

3- misol. $z_1=6\left(\cos 50°+i\sin 50°\right)$ va $z_2=2\left(\cos 25°+i\sin 25°\right)$ kompleks sonlar bo'linmasini toping.

Bo'lishning qoidasiga muvofiq:

$$\frac{z_1}{z_2}=\frac{6}{2}\left[\cos\left(50°-25°\right)+i\sin\left(50°-25°\right)\right]=3\left(\cos 25°+i\sin 25°\right).$$

Natural darajaga ko'tarish

$z=r\left(\cos\varphi+i\sin\varphi\right)$ kompleks sonni kvadratga ko'tarish uchun kompleks sonlarni ko'paytirish formulasi (1) dan foydalanamiz:

$$z^2=r^2\left(\cos\varphi+i\sin\varphi\right)\left(\cos\varphi+i\sin\varphi\right)=r^2\left(\cos 2\varphi+i\sin 2\varphi\right).$$

Huddi shundek, $z^3=\left[r\left(\cos\varphi+i\sin\varphi\right)\right]^3=r^3(\cos 3\varphi+i\sin 3\varphi)$. Umuman,

$z^n=\left[r\left(\cos\varphi+i\sin\varphi\right)\right]^n=r^n(\cos n\varphi+i\sin n\varphi)$ formula o'rinli, bunda $n\in N$.

4- misol. $z=3\left(\cos 15°+i\sin 15°\right)$ kompleks sonni kubga ko'taring:

(3) formulaga ko'ra:

$$z^3=27\left(\cos 45°+i\sin 45°\right)=\frac{27}{2}\left(\sqrt{2}+i\sqrt{2}\right)=\frac{27}{\sqrt{2}}(1+i).$$

5-misol. $z=\frac{1}{2}+\frac{\sqrt{3}}{2}i$ kompleks sonning 10- darajasini toping.

Avval berilgan sonning moduli va argumentini topib, uni trigonometrik ko'rinishda yozib olamiz: $r=\sqrt{\frac{1}{4}+\frac{3}{4}}=1$, $\varphi=\frac{\pi}{3}=60°$, $z=1\cdot\left(\cos 60°+i\sin 60°\right)$, bu yerdan:

$$z^{10} = (\cos 600° + i\sin 600°) = \cos 240° + i\sin 240° = -\frac{1}{2} - \frac{\sqrt{3}}{2}i .$$

Savol va topshiriqlar

1. Trigonometrik ko'rinishdagi kompleks sonlar qanday ko'paytiriladi? Ma'nosini oching va ayting.
2. Trigonometrik ko'rinishdagi kompleks sonlar qanday bo'linadi? Ma'nosini oching va ayting.
3. Trigonometrik ko'rinishdagi kompleks sonlar darajaga qanday ko'tariladi?

Mashqlar

Kompleks sonlarni ko'paytiring (**27–28**):

27. 1) $z_1 = \frac{\sqrt{3}}{2}(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$ va $z_2 = \frac{1}{2}(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$;

2) $z_1 = \frac{1}{3}(\cos\frac{\pi}{9} + i\sin\frac{\pi}{9})$ va $z_2 = 3(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3})$.

28. 1) $z_1 = \frac{1}{\sqrt{3}}(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12})$ va $z_2 = \sqrt{3}(\cos\pi + i\sin\pi)$;

2) $z_1 = 2(\cos\frac{\pi}{18} + i\sin\frac{\pi}{18})$ va $z_2 = \frac{1}{2}(\cos\frac{\pi}{2} + i\sin\frac{\pi}{2})$.

Kompleks sonlarni bo'ling (**29–30**):

29. 1) $z_1 = \sqrt{2}(\cos\frac{\pi}{8} + i\sin\frac{\pi}{8})$ ni $z_2 = 2(\cos\frac{\pi}{12} + i\sin\frac{\pi}{12})$ ga;

2) $z_1 = 8(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4})$ ni $z_2 = 4(\cos\frac{\pi}{6} + i\sin\frac{\pi}{6})$ ga.

30. 1) $z_1 = \cos\frac{3\pi}{4} + i\sin\frac{3\pi}{4}$ ni $z_2 = \cos\frac{\pi}{2} + i\sin\frac{\pi}{2}$;

2) $z_1 = \frac{\sqrt{3}}{2}(\cos\frac{5\pi}{3} + i\sin\frac{5\pi}{3})$ ni $z_2 = \frac{2}{\sqrt{3}}(\cos\frac{2\pi}{3} + i\sin\frac{2\pi}{3})$.

Kompleks sonni darajaga ko'taring (**31–32**):

31. 1) $(3(\cos\frac{\pi}{15} + i\sin\frac{\pi}{15}))^5$; 2) $(\sqrt{3}(\cos\frac{\pi}{3} + i\sin\frac{\pi}{3}))^6$; 3) $(\sqrt{2}(\cos\frac{\pi}{7} + i\sin\frac{\pi}{7}))^7$.

32. 1) $(4(\cos\frac{\pi}{4} + i\sin\frac{\pi}{4}))^4$; 2) $(\cos\frac{\pi}{15} + i\sin\frac{\pi}{15})^{10}$; 3) $(\cos\frac{\pi}{22} + i\sin\frac{\pi}{22})^{11}$.

Amallarni bajaring (**33–34**):

33. 1) $\dfrac{(1+i)^5\left(\sqrt{2}-i\right)^4}{(1-i)\left(1+\sqrt{2}\,i\right)^4}$; 2) $\dfrac{(1-i)^4\left(\sqrt{2}+i\right)^3}{(1+i)^4}$; 3) $\dfrac{(1+i)^{15}}{(1-i)^{10}-(1+i)^{10}\cdot i}$.

34. 1) $\dfrac{2+5i}{2-5i}+\dfrac{2-5i}{2+5i}$; 2) $\dfrac{12+5i}{6-8i}+\dfrac{(2-i)^2}{1-2i}$; 3) $\dfrac{3-4i}{3+4i}+\dfrac{5+6i}{5-6i}$.

91 KOMPLEKS SONDAN KVADRAT ILDIZ CHIQARISH

$z=r(\cos\varphi+i\sin\varphi)$ korinishdagi kompleks sondan kvadrat ildiz chiqarish uchun izlanayotgan kompleks sonning modulini x va argumentini y deb quyidagi tenglikni yozamiz:

$$\sqrt{r(\cos\varphi+i\sin\varphi)}=x(\cos y+i\sin y).$$

Tenglikning ikkala qismini kvadratga ko‘tarib, $r(\cos\varphi+i\sin\varphi)=x^2(\cos 2y+i\sin 2y)$ hamda $x^2=r$, $2y=\varphi+2\pi n$ ekanidan $x=\sqrt{r}$, $y=\dfrac{\varphi}{2}+\pi n$, $n\in\mathbb{Z}$ munosabatlarni topamiz. Demak, izlanayotgan z kompleks sonning kvadrat ildizi uchun

$$\beta=\sqrt{r}\left[\cos\frac{\varphi+2\pi n}{2}+i\sin\frac{\varphi+2\pi n}{2}\right]$$

formula o‘rinli. n ga $0,\ \pm1,\ \pm2,\ \ldots$ qiymatlarni qo‘yib, turli ildizlarni topamiz. Tekshirish natijasida faqat 2 ta turli qiymat borligi aniqlanadi, ya’ni

$$n=0 \text{ da } \beta_1=\sqrt{r}\left(\cos\frac{\varphi}{2}+i\sin\frac{\varphi}{2}\right), \qquad (1)$$

$$n=1 \text{ da } \beta_2=\sqrt{r}\left[\cos\left(\frac{\varphi}{2}+\pi\right)+i\sin\left(\frac{\varphi}{2}+\pi\right)\right]. \qquad (2)$$

1- misol. $z=9(\cos 60^\circ+i\sin 60^\circ)$ kompleks sondan kvadrat ildiz chiqaring.

△ Yuqoridagi formulaga ko‘ra, kvadrat ildizlarni hisoblaymiz:

$$\sqrt{z}=3\left[\cos(30^\circ+180^\circ n)+i\sin(30^\circ+180^\circ n)\right].$$

Bu formulada

$n=0$ uchun $\sqrt{z}=3(\cos 30^\circ+i\sin 30^\circ)=\dfrac{3}{2}\left(\sqrt{3}+i\right)$ va

$n=1$ uchun $\sqrt{z}=3(\cos 210^\circ+i\sin 210^\circ)=-\dfrac{3}{2}\left(\sqrt{3}+i\right)$ kvadrat ildizlar topiladi. ▲

Kompleks sondan kub ildiz chiqarishda quyidagi formuladan foydalaniladi:

$$z_n=\sqrt[3]{r(\cos\varphi+i\sin\varphi)}=x(\cos y+i\sin y)=\sqrt[3]{r}(\cos\frac{\varphi+360^\circ n}{3}+i\sin\frac{\varphi+360^\circ n}{3}),$$

$n=0,\ 1,\ 2.$

Bu topilgan sonlar Dekart koordinatalar sistemasida markazi koordinata boshida va radiusi $\sqrt[3]{r}$ bo'lgan aylanaga ichki chizilgan muntazam uchburchak uchlaridan iboratdir.

2- misol. $z=1$ ko'mpleks sonning kub ildizini toping va chizmada ko'rsating.

△ Bu sonning moduli $r=1$ va argumenti $\varphi=0^\circ$ bo'lgani uchun,

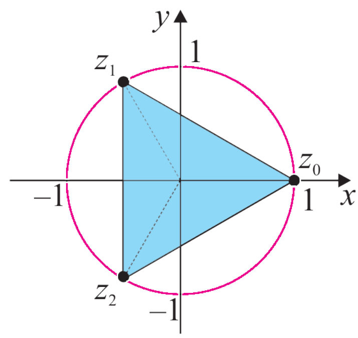
$$z_n=\sqrt[3]{1}=1\cdot\left(\cos\frac{0^\circ+360^\circ n}{3}+i\sin\frac{0^\circ+360^\circ n}{3}\right),\ n=0,\ 1,\ 2.$$

Bu yerdan: $n=0$ da $z_0=1\cdot(\cos 0^\circ+i\sin 0^\circ)=1,$

$n=1$ da $z_1=1\cdot(\cos 120^\circ+i\sin 120^\circ)=-\frac{1}{2}+\frac{\sqrt{3}}{2}i,$

$n=2$ da $z_2=1\cdot(\cos 240^\circ+i\sin 240^\circ)=-\frac{1}{2}-\frac{\sqrt{3}}{2}i.$

Bu sonlar muntazam uchburchakning uchlaridan iborat ekanini 5- rasmdan ko'rishimiz mumkin.



5-rasm.

Kompleks sondan 4- darajali ildiz chiqarishda quyidagi formuladan foydalaniladi:

$$z_n=\sqrt[4]{r(\cos\varphi+i\sin\varphi)}=x(\cos y+i\sin y)=\sqrt[4]{r}\left(\cos\frac{\varphi+360^\circ n}{4}+i\sin\frac{\varphi+360^\circ n}{4}\right),$$

$n=0,\ 1,\ 2,\ 3.$

3- misol. $z=i$ kompleks sondan 4- darajali ildiz chiqaring.

△ Bu sonning moduli $r=1$, argumenti $\varphi=90^\circ$ bo'lgani uchun

$$z_n=\sqrt[4]{1(\cos 90^\circ+i\sin 90^\circ)}=1\cdot\left(\cos\frac{90^\circ+360^\circ n}{4}+\sin\frac{90^\circ+360^\circ n}{4}\right).$$

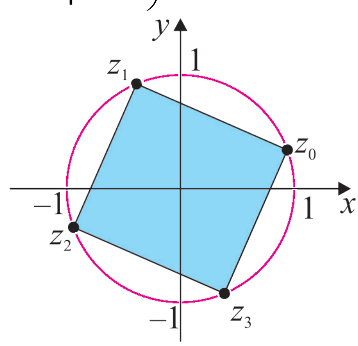
Bu yerdan: $n=0$ da $z_0=\cos 22{,}5^\circ+i\sin 22{,}5^\circ,$

$n=1$ da $z_1=\cos 112{,}5^\circ+i\sin 112{,}5^\circ,$

$n=2$ da $z_2=\cos 202{,}5^\circ+i\sin 202{,}5^\circ,$

$n=3$ da $z_3=\cos 292{,}5^\circ+i\sin 292{,}5^\circ.$

Bu sonlar markazi koordinata boshida va radiusi 1 bo'lgan aylanara ichki chizilgan kvadratning uchlaridan iboratdir (6- rasm).



6-rasm

Savol va topshiriqlar

1. Kompleks sondan kvadrat ildiz qaysi formula orqali topiladi?
2. Muavr formulasi nima? Uning ma'nosini oching va ayting.

Mashqlar

Kompleks sondan kvadrat ildiz chiqaring (**35–36**):

35. 1) $z=25\left(\cos\frac{\pi}{3}+i\cdot\sin\frac{\pi}{3}\right)$; 2) $z=\frac{1}{4}\left(\cos\frac{\pi}{18}+i\cdot\sin\frac{\pi}{18}\right)$;

3) $z=\cos\frac{\pi}{5}+i\cdot\sin\frac{\pi}{5}$; 4) $z=\cos\frac{3\pi}{4}+i\cdot\sin\frac{3\pi}{4}$;

5) $z=2\left(\cos\frac{\pi}{30}+i\cdot\sin\frac{\pi}{30}\right)$; 6) $z=\frac{1}{49}\left(\cos\frac{\pi}{8}+i\cdot\sin\frac{\pi}{8}\right)$;

7) $z=\cos\frac{\pi}{10}+i\cdot\sin\frac{\pi}{10}$; 8) $z=\cos\frac{3\pi}{2}+i\cdot\sin\frac{3\pi}{2}$;

36. 1) $z=2\left(\cos\frac{\pi}{3}+i\cdot\sin\frac{\pi}{3}\right)$; 2) $z=\frac{1}{2}\left(\cos\frac{\pi}{8}+i\cdot\sin\frac{\pi}{8}\right)$;

3) $z=\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4}$; 4) $z=\cos\frac{3\pi}{2}+i\cdot\sin\frac{3\pi}{2}$;

5) $z=2\left(\cos\frac{\pi}{2}+i\cdot\sin\frac{\pi}{2}\right)$; 6) $z=\frac{16}{9}\left(\cos\frac{\pi}{8}+i\cdot\sin\frac{\pi}{8}\right)$;

7) $z=5(\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4})$; 8) $z=\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4}$.

IV bobga doir mashqlar

37. Hisoblang:

1) $(3+4i)(2-5i)+(3-4i)(2+5i)$; 2) $(1+3i)^3-(4+i^5)$;

3) $\frac{(1-2i)^2}{1+3i}$; 4) $5-7i+8i^2-9i^3+i^4$.

38. Algebraik ko'rinishda yozing:

1) $z=\left(\frac{1-\sqrt{3}\,i}{3i}\right)^2$; 2) $z=\frac{12-13i}{8+6i}+\frac{(1+2i)^2}{i+3}$; 3) $\frac{4i}{(\sqrt{3}-i)^2}$.

Hisoblang (**39–42**):

39. 1) $(1+i)^{10}$; 2) $(1-i)^4\left(-2\sqrt{3}+2i\right)^3$; 3) $(1+i)^{2018}\cdot(1-1)^{2018}$;

4) $\left(\dfrac{\sqrt{3}+i}{1-i}\right)^{8}$; | 5) $\dfrac{2\sqrt{3}-2i}{(-1+i)\left(\sqrt{2}+\sqrt{6}\,i\right)}$; | 6) $\left(\dfrac{\sqrt{2}-i}{1+i}\right)^{10}$.

40. 1) $z=\dfrac{(2+i)^2}{3-4i}$; | 2) $z=\dfrac{(1+2i)^3}{2i}-3i^{10}$; | 3) $z=\left(\dfrac{1-i}{1+i}\right)^5$;

4) $z=\dfrac{3+2i}{1+4i}-i^7$; | 5) $\dfrac{(4-i)}{3+4i}$; | 6) $\dfrac{2-3i}{1-4i}$.

41. 1) $\dfrac{2+5i}{2-5i}+\dfrac{2-5i}{2+5i}$; | 2) $\dfrac{12+5i}{6-8i}+\dfrac{(2-i)^2}{1-2i}$;

3) $(2-3i)^3-(2+3i)^3$; | 4) $\dfrac{(4+3i)(2+3i)^2}{6+8i}$;

5) $\dfrac{33+5i}{2-5i}+\dfrac{2-5i}{2+5i}$; | 6) $\dfrac{12-5i}{6-8i}+\dfrac{(2+i)^2}{1-2i}$.

42. 1) $(2-2i)\cdot 2\sqrt{3}\left(\cos 70^\circ+i\sin 70^\circ\right)$; | 2) $\left(\cos\left(-\dfrac{\pi}{6}\right)+i\sin\left(-\dfrac{\pi}{6}\right)\right)\cdot\left(\sqrt{3}-3i\right)$.

43. Bo‘lishni bajaring:

1) $5\left(\cos 100^\circ+i\sin 100^\circ\right):\left(\dfrac{\sqrt{3}}{2}+\dfrac{1}{2}i\right)$; | 2) $(6+6i):3\left(\cos 75^\circ+i\sin 75^\circ\right)$.

44. Darajaga ko‘taring:

1) $\left(1-\sqrt{3}i\right)^3$; | 2) $\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)^4$; | 3) $\left(\dfrac{1}{\sqrt{3}}+\dfrac{1}{\sqrt{3}}i\right)^6$; 4) 2 2 ;

4) $\left(1-\sqrt{3}i\right)^5$; | 5) $\left(\cos\dfrac{3\pi}{4}+i\sin\dfrac{3\pi}{4}\right)^{10}$; | 6) $\left(\dfrac{1}{\sqrt{3}}+\dfrac{1}{\sqrt{3}}i\right)^{10}$. 8) 2 2 .

45. Kvadrat ildizni hisoblang:

1) $\sqrt{-27i}$; 2) $\sqrt{6-6\sqrt{3}\,i}$; | 3) $\sqrt{8+8\sqrt{3}\,i}$; | 4) $\sqrt{-256}$.

46. Tenglikni tekshiring:

1) $\left[\dfrac{-\sqrt{3}+i}{2}\right]^5+\left[\dfrac{-\sqrt{3}-i}{2}\right]^5=\sqrt{3}$;

2) $\dfrac{(\sin 26^\circ+i\cos 154^\circ)\cdot(\sin 27^\circ+i\cos 153^\circ)^3}{\sin 17^\circ-i\cos 17^\circ}=-1$.

47. Kub ildizni hisoblang:

1) $\sqrt[3]{1+i}$; | 2) $\sqrt[3]{-i}$; | 3) $\sqrt[3]{8}$; | 4) $\sqrt[3]{1-i}$; | 5) $\sqrt[3]{-8}$.

48. 4- darajali ildiz chiqaring:

1) $\sqrt[4]{-1}$; | 2) $\sqrt[4]{16}$; | 3) $\sqrt[4]{1+i}$; | 4) $\sqrt[4]{1-i}$; | 5) $\sqrt[4]{-16}$.

Nazorat ishi namunalari

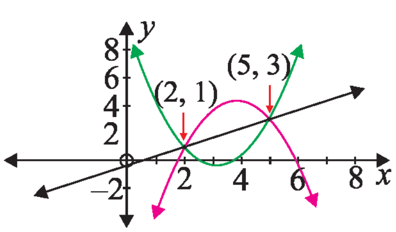
1. Hisoblang: $(35-7i)\cdot(4-6i)$.
2. Bo‘lishni bajaring: $\dfrac{8-i}{40+3i}$.
3. Ko‘paytiring: $3(\cos5°+i\sin5°)\cdot8(\cos3°+i\sin3°)$.
4. Darajaga ko‘taring: $(3(\cos4°+i\sin4°))^6$
5. Kvadrat ildiz chiqaring: $\sqrt{64i}$.

JAVOBLAR

III bob

73. a) Barcha x abssissalar turli bo‘lgani uchun bu funksiya bo‘ladi; b) ikkita nuqtada x abssissalar bir hil bo‘lgani uchun bu funksiya bo‘lmaydi; c) barcha nuqtalarda x abstsissalar bir hil bo‘lgani uchun bu funksiya bo‘lmaydi; d) barcha x abssissalar turli bo‘lgani uchun bu funksiya bo‘ladi; e) barcha x abssissalar turli bo‘lgani uchun bu funksiya bo‘ladi; f) barcha nuqtalarda x abssissalar bir hil bo‘lgani uchun bu funksiya bo‘lmaydi. **74.** a) Funksiya; b) funksiya; c) funksiya; d) funksiya emas; e) funksiya; f) funksiya emas; g) funksiya; h) funksiya emas. **75.** Yo‘q, har qanday vertikal to‘g‘ri chiziq funksiya bo‘lmaydi. **76.** Yo‘q, $y=\pm\sqrt{9-x^2}$. **77.** a) 2; b) 8; c) –1; d) –13; e) 1. **78.** a) 2; b) 2; c) –16; d) –68; e) $\frac{17}{4}$ **79.** a) –3; b) 3; c) 3; d) –3; e) $\frac{15}{2}$. **80.** a) $7-3a$; b) $7+3a$; c) $-3a-2$; d) $10-3b$; e) $1-3x$; f) $7-3x-3h$. **81.** a) $2x^2+19x+43$; b) $2x^2-11x+13$; c) $2x^2-3x-1$; d) $2x^4+3x2-1$; e) $2x^4-x^2-2$; f) $2x^2+4hx+2h^2+3x+3h-1$. **82.** a) **I)** $-\frac{7}{2}$; **II)** $-\frac{3}{4}$; **III)** $-\frac{4}{9}$; b) $x=4$. **84.** $V(4)=6210$. Bu uskunaning 4 yildan so‘ng bo‘ladigan narxi. $t=4{,}5$ shuncha yildan keyin uskunaning narxi 5780 bo‘ladi. Uskunaning dastlabki narxi 9650 ga teng.

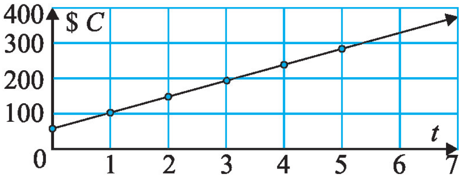
85.



86. $f(x)=-2x+5$. **87.** $a=3$, $b=-2$. **88.** $a=3$, $b=-1$, $c=-4$. **90.** a) **I)** $x>0$; b) **II)** $-2\le x\le3$; c) **I)** $-2<x\le0$; **II)** $0\le x<2$; d) **I)** $x\le2$; **II)** $x\ge2$; e) **II)** $x\in\mathbb{R}$; f) **I)** $x\in\mathbb{R}$; g) **I)** $1\le x\le5$; **II)** $x\le1$, $x\ge5$; h) **I)** $2\le x<4$, $x>4$; **II)** $x<0$, $0<x\le2$; i) **I)** $x\le0$, $2\le x\le6$; **II)** $0\le x\le2$, $x\ge6$. **92.** a) V(0)=25000 yevro. Bu avtomashinaning boshlang‘ich narxi; b) $V(3)=16\,000$. Bu avtomashinaning 3 yildan so‘ng bo‘lgan narxi; c) $t=5$.

93. a)

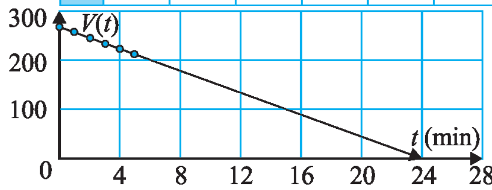
t	0	1	2	3	4	5
C	60	105	150	195	240	285



b) $C=60+45t$; c) $ 352,50.

94. a)

t	0	1	2	3	4	5
V	265	254	243	232	221	210

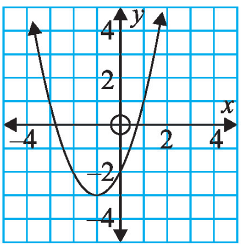
b) 

c) $V(t) = 265–11t$; d) **I)** 100 l.

95. a) Ha; b) yo‘q; c) ha; d) ha; e) ha; f) yo‘q. **96.** a) Yo‘q; b) ha; c) ha; d) ha; e) yo‘q; f) yo‘q. **97.** a) $x=–3$; b) $x=–2$ yoki $–3$; c) $x=1$ yoki 4; d) haqiqiy yechimga ega emas. **98.** a) **I)** 75 m; **II)** 195; **III)** 275 m; b) **I)** $t=2$ s yoki t=14 s; **II)** $t=0$ s yoki $t=16$ s. **99.** a) 40 ming, 480 ming; b) 10 ta yoki 62 ta.

100. a)

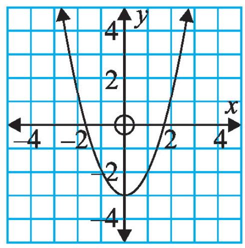
x	–3	–2	–1	0	1	2	3
y	1	–2	–3	–2	1	6	13



$y=x^2+2x–2$

b)

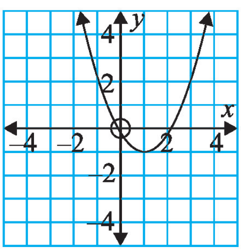
x	–3	–2	–1	0	1	2	3
y	6	1	–2	–3	–2	1	6



$y=x^2–3$

c)

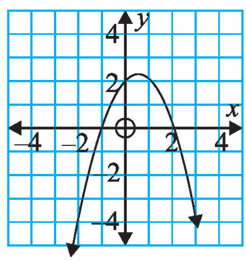
x	–3	–2	–1	0	1	2	3
y	15	8	3	0	–1	0	3



$y=x^2–2x$

d)

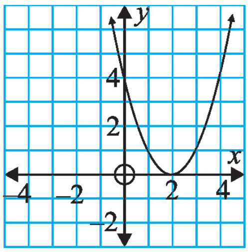
x	–3	–2	–1	0	1	2	3
y	–10	–4	0	2	2	0	–4



$f(x)=–x^2+x+2x$

e)

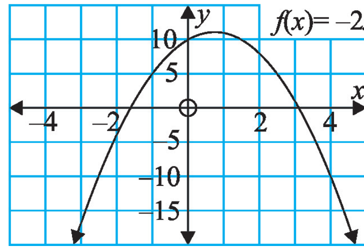
x	–3	–2	–1	0	1	2	3
y	25	16	9	4	1	0	1



$y=x^2–4x+4$

f)

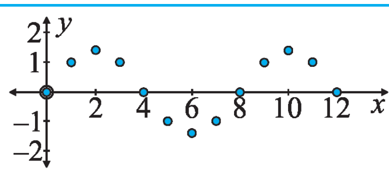
x	–3	–2	–1	0	1	2	3
y	–17	–4	5	10	11	8	1



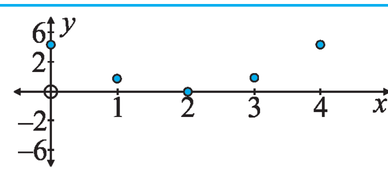
$f(x)= –2x^2+3x+10$

101. a) 3; b) –1; c) –4; d) 1; e) 5; f) 0; g) 8; h) –5; i) 2. **102.** a) 3; b) –6; c) 49; d) 15; e) 0; f) 20. **105.** a) $x=3$; b) $x=–5/2$; c) $x=1$; d) $x=–4$; e) $x=3$; f) $x=–4$. **106.** a) $x=4$; b) $x=–2$; c) $x=1$; d) $x=11/2$; e) $x=5$; f) $x=–2$. **107.** a) $x=–3$; b) $x=4$; c) $x=–5/4$; d) $x=3/2$; e) $x=0$; f) $x=7/10$; g) $x=3$; h) $x=5/3$; i) $x=–4$. **108.** a) (2, 3); b) (–1, 4); c) (3, 8); d) (0, 3); e) (–3, –18); f) (1, –1); g) (1/2, –5/4); h) (3/4, –7/8); i) (6, 7). **109.** a) $y=2(x–1)(x–2)$; b) $y=2(x–2)^2$; c) $y=(x–1)(x–3)$; d) $y=–(x–3)(x+1)$; e) $y=–3(x–1)^2$; f) $y=–2(x+2)(x–3)$. **110.** a) $y=3/2(x–2)(x–4)$; b) $y=–1/2(x+4)(x–2)$; c) $y=–4/3(x+3)^2$; d) $y=1/4(x+3)(x–5)$; e) $y=–(x+3)(x–3)$; f) $y=4(x–1)(x–3)$. **111.** a) 3m; b) 0,5s; c) 4m.

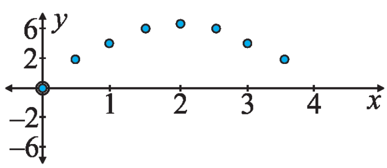
113. a) davriy;



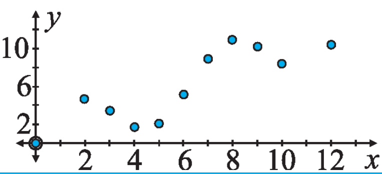
b) davriy emas;



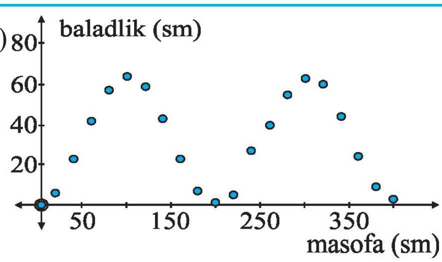
c) davriy emas;



d) davriy emas;



114. a)

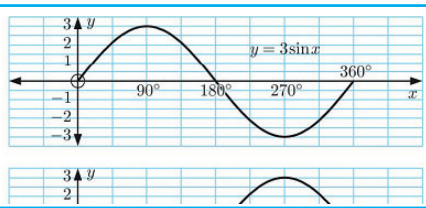


b) O‘q tenglamasi maksimum davr amplituda mos ravishda $y=32$; 64 sm; 200 sm; 32 sm ga teng.

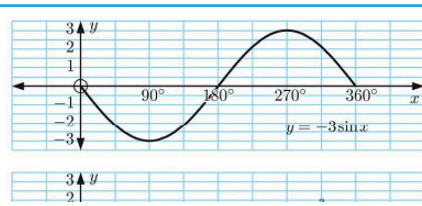
115. a) davriy; b) davriy; c) davriy; d) davriy emas; e) davriy; f) davriy.

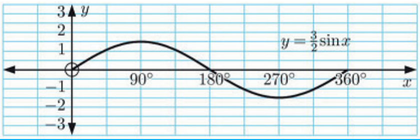
116. a) 2; b) 8; c) (2,1); d) 8; e) $y=–1$.

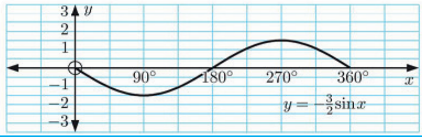
117. a)

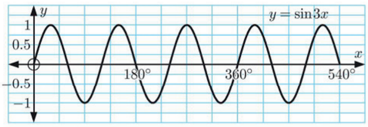


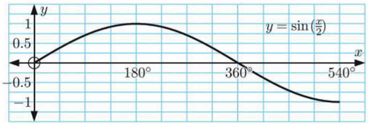
b)

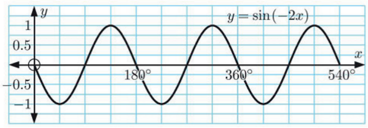


c) 

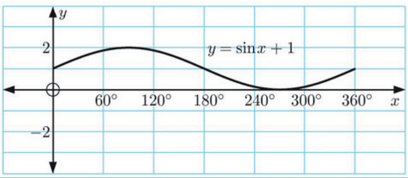
d) 

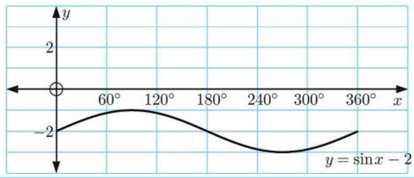
118. a) 

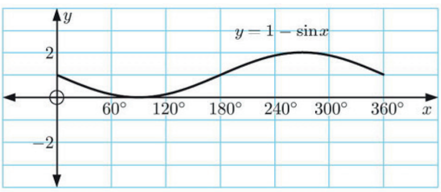
b) 

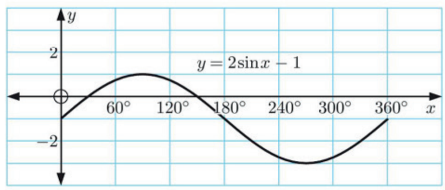
c) 

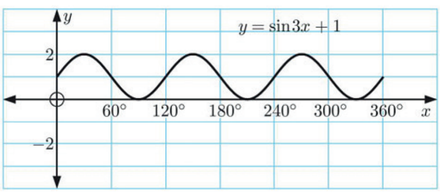
119. a) 90°; b) 90°; c) 1080°; d) 600°; **120.** a) b=2/5; b) b=3; c) b=1/6.
121. a) y=3sin$x$; b) y=sin$x$–2; c) y=–2sin$x$–1; d) y=sin2x; e) y=–4sin(x/2); f) y=sin(x/2); g) y=2sin3x; h) y=2sin2x–3.

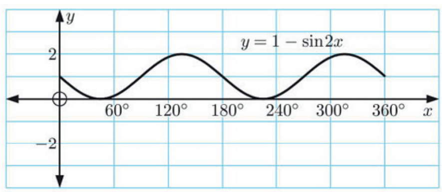
122. a) 

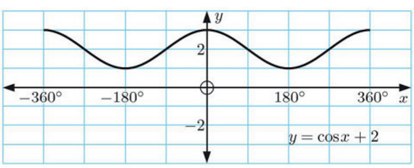
b) 

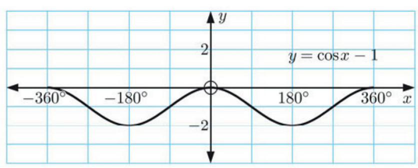
c) 

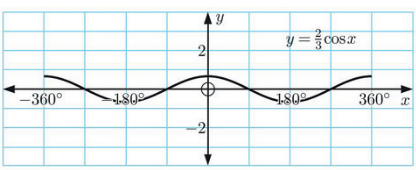
d) 

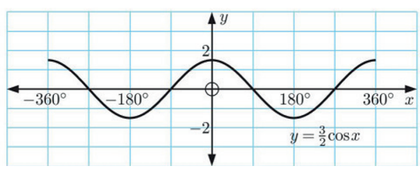
e) 

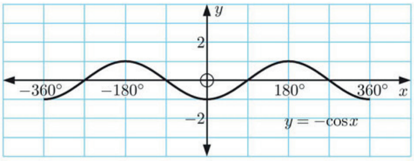
f) 

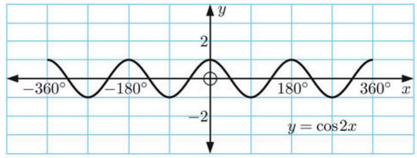
123. a) 

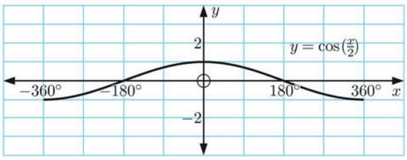
b) 

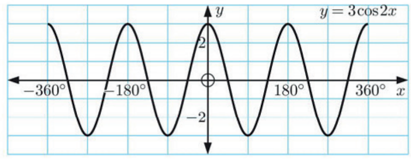
c) 

d) 

e) 

f) 

g) 

h) 

124. a) 120°; b) 1080°; c) 720°. **126.** a) $y=2\cos 2x$; b) $y=\cos(x/2)+2$; c) $y=-5x\cos 2x$. **127.** $T=9{,}5\cos(30t)-9{,}5$. **130.** 1) 0; 2) $\frac{\pi}{3}$; 3) $\frac{\pi}{6}$; 4) $-\frac{\pi}{3}$. **131.** 1) $-\frac{\pi}{4}$; 2) $-\frac{\pi}{6}$; 3) $\frac{\pi}{2}$; 4) $-\frac{\pi}{2}$. **132.** 1) $\frac{\pi}{2}$; 2) $\frac{5\pi}{6}$; 3) $\frac{\pi}{4}$; 4) π. **136.** 1) 0; 2) $\frac{4\pi}{3}$. **138.** 1) $\frac{3\pi}{2}$; 2) –π. **140.** 1) 2π; 2) $\frac{3\pi}{2}$. **142.** 1) ma’noga ega; 2) ma’noga ega emas; 3) ma’noga ega emas.

144. 1) $x=(-1)^{n+1}\frac{\pi}{6}+n\pi,\ n\in Z.$ **146.** 1) $x=\pm\frac{3\pi}{4}+2n\pi,\ n\in Z.$

148. 1) $x=-\frac{\pi}{3}+n\pi,\ n\in Z.$ **150.** 1) $x=\pm\frac{2\pi}{3}+2n\pi,\ n\in Z.$

151. 1) $x=(-1)^{n}\frac{\pi}{4}+n\pi,\ n\in Z.$ **152.** 2) $x=-\frac{\pi}{24}+\frac{n\pi}{4},\ n\in Z.$

153. 1) $x_1=k\pi,\ x_2=\frac{\pi}{4}+k\pi,\ k\in Z.$

156. 1) $x_1=(-1)^{n+1}\frac{\pi}{4}+n\pi,\ x_2=-\frac{\pi}{8}+\frac{n\pi}{2},\ n\in Z.$

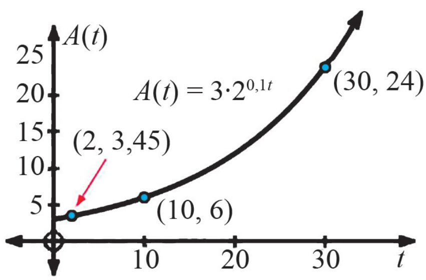
157. 1) $x_1=-\frac{\pi}{2}+2n\pi,\ \ x_2=(-1)^{n}\frac{\pi}{6}+n\pi,\ n\in Z.$

158. 2) $x=\pm\arccos(1-\frac{\sqrt{7}}{2})+2n\pi,\ n\in Z.$ **159.** 2) $x_1=-\frac{\pi}{4}+n\pi,$ $x_2=\arccos 4+n\pi,\ n\in Z.$ **160.** 1) $x=\frac{2n\pi}{3},\ n\in Z;$ 3) $x_1=\frac{n\pi}{2},$ $x_2=\frac{\pi}{10}+\frac{n\pi}{5},\ n\in Z.$ **162.** 1) $\left(\frac{\pi}{6};\ \frac{5\pi}{6}\right)$; 2) $\left(-\frac{\pi}{4};\ \frac{\pi}{4}\right)$; 3) $\left(-\frac{\pi}{3};\ \frac{\pi}{2}\right)$.

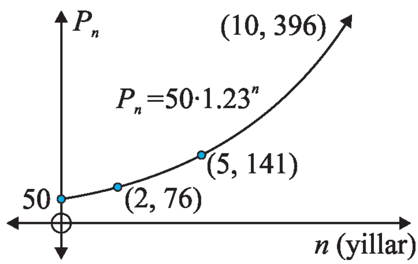
163. 1) $\left[\frac{\pi}{4}+2n\pi;\ \frac{3\pi}{4}+2n\pi\right],\ n\in Z;$ 2) $\left(\frac{3\pi}{4}+2n\pi;\ \frac{5\pi}{4}+2n\pi\right),\ n\in Z;$

3) $\left(-\frac{\pi}{2}+n\pi;\ -\frac{\pi}{4}+n\pi\right)$, $n \in Z$. **167.** 1) $\left[-\frac{\pi}{2}+n\pi;\ \frac{\pi}{3}+n\pi\right]$, $n \in Z$. **173.** 1) $y=2x+6$. **174.** 1) $y=13\cdot\sqrt{\frac{x-1}{17}}$. **175.** 1) $x^2+y^2=49$, aylana. **176.** 1) $(x-3)^2+(y-7)^2=36$, aylana. **177.** 1) 3; 2) 1; 3) 4; 4) 4. **178.** 1) katta; 2) kichik. **180.** 1) aniqlanish sohasi: $(-\infty;+\infty)$, qiymatlar sohasi: $(0;\ +\infty)$, $(-\infty;\ +\infty)$ oraliqda oʻsadi. **181.** 1) oʻsadi; 2) kamayadi; 3) oʻsadi. **183.** 1) $(-\infty;\ 1]$; 2) $\left(-\infty;\frac{4}{9}\right)$; 7) $[1;\ +\infty]$; 12) $\left(-\infty;\ -2-\sqrt{34}\right)\cup\left(-2+\sqrt{34};\ +\infty\right)$. **184.** 1) $(-\infty;\ 2]$. **185.** 1) 3; 2) –2; 3) –2; 4) –3; 5) –3. **186.** 1) katta; 2) katta; 3) kichik. **187.** 1) 2; 2) 5; 3) 125; 4) 45; 5) $\frac{1}{36}$; 9) –2. **188.** 1) $(2,5;\ +\infty)$; 2) $(-\infty;-1)\cup(3;+\infty)$; 3) $(-2;2)$. **190.** 1) $\frac{1}{32}$; 2) 1; 3) 4; 4) 2; 8) –2; 10) 0,5 va 1; 15) $\frac{1}{7}$ va 49. **191.** 1) $(64;\ +\infty)$; 2) $\left(0;\ \frac{1}{3}\right)\cup(27;+\infty)$; 7) $(2;5)$.

192. a) 3 m²;
b) **I)** 3,45 m²; **II)** 6 m²; **III)** 24 m²;
c)

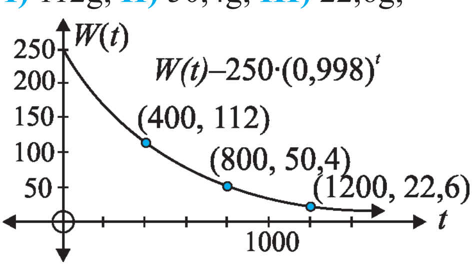


193. a) 50;
b) **I)** 76; **II)** 141; **III)** 396;
c)



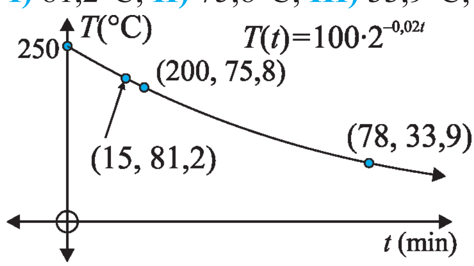
194. a) V_0; b) $2V_0$; c) 100%; d) 183 foizga ortadi.

195. a) 250g;
b) **I)** 112g; **II)** 50,4g; **III)** 22,6g;
c)

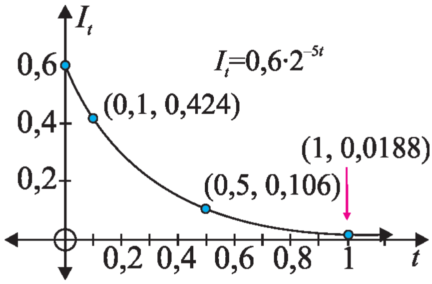


d) ≈346.

197. a) 100°C;
b) **I)** 81,2°C; **II)** 75,8°C; **III)** 33,9°C;
c)



198. a) 0,6 amper;
b) **I)** 0,424 amper; **II)** 0,106 amper;
III) 0,0188 amper;
c)



199. a) L_0; b) 99%. **200.** Taxminan 3 soat 15 min. **201.** 37,8 oy. **202.** 10,8 minut. **203.** 22,7 yil. **204.** b) 1. **205.** a) $\{-14;\ 3;\ 15\}$; c) $\{-4;4\}$. **206.** a) $\left(-\infty;-\frac{1}{3}\right)\cup\left(\frac{3}{2};2\right)$.
207. a) $(-\infty;-6]\cup[-2;+\infty)$.
208. a) $\left(-2;\frac{-1-\sqrt{5}}{2}\right)\cup\left(\frac{-1+\sqrt{5}}{2};+\infty\right)$;
c) $(-\infty;-2)\cup[1;+\infty)$.

209. a) $\{(7;8);(8;7);(-7;-8);+8;-7)\}$. **212.** a) 3; b) 2. **213.** a) kichik; b) kichik. **216.** a) $(-\infty;-1]\cup[1;+\infty)$; b) $(-\infty;+\infty)$. **217.** a) $(0;1]$; b) $(3;+\infty)$; c) $(-\infty;\ 0)$. **218.** a) $\frac{1}{15}$; b) 0 va 1; c) 1 va –2. **219.** c) 0. **220.** a) $\{(2;3);(-3;8)\}$. **221.** a) $(-\infty;0]$; b) $(-\infty;1{,}5)$. **222.** a) kichik; b) katta. **223.** a) $(-3{,}5;\ +\infty)$; b) $(-2;2)$. **224.** a) $2\sqrt{5}$. **225.** b) $(100000;0{,}1)$. **226.** a) $(3;1)$. **227.** a) $(-\infty;-2]\cup[1;+\infty)$. **229.** a) kichik; b) katta. **230.** a) $-\frac{2\pi}{3}$ **231.** c) $x_1=\frac{\pi}{4}+n\pi,\ x_2=\arccos\frac{1}{4}+n\pi,\ n\in Z.$
234. a) $\left(-\frac{\pi}{6}+2n\pi;\frac{7\pi}{6}+2n\pi\right),\ n\in Z.$ **235.** c) $\left(-\frac{\pi}{18}+\frac{\pi n}{9};\frac{\pi}{27}+\frac{n\pi}{9}\right),\ n\in Z.$

IV bob

1. 7) Re(z)=–7, Jm(z)=3; 8) Re(z)=8, Jm(z)=5; 9) Re(z)=–0,5, Jm(z)=–6; 10) Re(z)=–5,7, Jm(z)=–5; 11) Re(z)=0, Jm(z)=–5; 12) Re(z)=90, Jm(z)=0.
6. 1) $\bar{z}=7{,}2$; 3) $\bar{z}=4+3i$. **8.** 1) 16; 3) $3+i$. **10.** 1) $8i$; 2) $-1-5i$; 3) $-3+i$. **12.** 2) $1\frac{1}{6}-\frac{1}{6}i$. **14.** 1) $-\frac{23}{13}-\frac{2}{13}i$; 3) $\frac{5}{3}-\frac{2}{3}i$. **16.** 2) $\frac{12}{13}$. **20.** 1) 10; 2) 4; 3) 2; 4) 2. **22.** 1) 0; 2) $\frac{3\pi}{2}$; 3) $\frac{11\pi}{6}$. **24.** 1) $2\left(\cos\frac{7\pi}{4}+i\cdot\sin\frac{7\pi}{4}\right)$ va $2\cdot e^{\frac{7\pi i}{4}}$.
28. 1) $z_1\cdot z_2=\cos\frac{13\pi}{12}+i\cdot\sin\frac{13\pi}{12}$. **30.** 1) $\cos\frac{\pi}{4}+i\cdot\sin\frac{\pi}{4}$. **32.** 2) $\cos\frac{2\pi}{3}+i\cdot\sin\frac{2\pi}{3}$.
34. 1) $-\frac{42}{29}$; 2) $-18i$. **36.** 1) $z_0=\sqrt{2}(\cos\frac{\pi}{6}+i\cdot\sin\frac{\pi}{6})$, $z_1=\sqrt{2}(\cos\frac{7\pi}{6}+i\cdot\sin\frac{7\pi}{6})$.

Foydalanilgan va tavsiya etiladigan adabiyotlar

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